K.S.R.M. COLLEGE OF ENGINEERING

(UGC-AUTONOMOUS)

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Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu.

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MATHS

1. Let z= x + iy $\Rightarrow \frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$

Given $\text{Im}\left(\frac{2z+1}{iz+1}\right) = -4 \Rightarrow \frac{2y(1-y)-x(2x+1)}{x^2+(1-y)^2} = -4$

$x^2 + y^2 - x/2 - 3y + 2 = 0$

2. $x^2 - 3x + 5 = 0$, α, β are the roots

$\alpha + \beta = 3$ *and* $\alpha\beta = 5$

Now $\frac{5\alpha\beta^{2016} + 3\alpha^{2017}.\beta + \alpha\beta^{2018}}{\alpha^{2016} + \beta^{2016}} = \alpha\beta(\alpha+\beta) = 3\times5 = 15$

3.. Put x=0 in the L.H.S

Then we get skew symmetric matrix

Determinant of skew symmatrix is zero

4. $|adj\ A| = |A|^{n-1}$ here n=3, $|A| = 6$

$= 6^2 = 36$

$$\begin{vmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & P & 0 \end{vmatrix} = 36 \Rightarrow P = 2$$

5. $\left[\frac{200}{2}\right] + \left[\frac{200}{2^2}\right] + \left[\frac{200}{2^3}\right] + ... + \left[\frac{200}{2^7}\right] \Rightarrow 100 + 50 + 25 + 12 + 6 + 3 + 1 = 197$

6.

$CC \rightarrow 4! = 24$

$CE \rightarrow 4! = 24$

$CIC \rightarrow 3! = 6$

$CIE \rightarrow 3! = 6$

$CIL \rightarrow 3! = 6$

$CIRCEL \rightarrow 1 = 1$

$CIRCLE \rightarrow 01 = 1$

$---$

68

7. Total no. of balls=30

No of balls that are either white or black or green=11+7+5=23

Required probability=23/30

8. $T_{r+1} = 9_{C_r}\left(\frac{k^2}{2}\right)^{9-r}\left(\frac{-1}{x}\right)^r \Rightarrow 9_{C_r}.\frac{x^{18-3r}}{2^{9-r}}(-1)^r$

Here 18-3r = 0,r = 6

$k = \frac{9_{C_6}}{2^3} = \frac{9\times8\times7}{6\times8} = \frac{21}{2} \Rightarrow 2k = 2\times\frac{21}{2} = 21$

9. For rational term r20, 3, 6, 9, 12, 15, 18(r should be multiple of 3) and 'n' should be even because $\frac{18-r}{2} \in I$

R= 0,6,12,18 , Total no of rational terms =4

10. $16\sin 20^0 \sin 40^0 \sin 80^0 \Rightarrow 16\sin 20^0 \sin(60^0 - 20^0)\sin(60^0 + 20^0)$

$16\times\frac{1}{4}\sin 3(20^0) \Rightarrow \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

11. $s(n): 10^n + 3(4^{n+2}) + 5 \Rightarrow s(1): 10 + 192 + 5 = 207(divisible\ by\ 3)$

$s(2): 100 + 3(256) + 5 = 873(divisible\ by\ 3)$

$s(n)\ is\ divisible\ by\ 3$

12. $(1+\omega-\omega^2)^7 = \left(-\omega^2 - -\omega^2\right)^7 = (-2\omega^2)^7 = -128\omega^2$

13. $\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]^{3/4} = \left(c\,i\,s\,\pi\right)^{1/4} \Rightarrow cis\left(\frac{16\pi}{4}\right) = 1$

14. Area of triangle = $\frac{1}{2}|z|^2 = 50 \Rightarrow |z| = 10$

15. Let $y = \frac{x}{x^2 - 5x + 9} \Rightarrow -11y^2 + 10y + 1 \geq 0 \Rightarrow -\frac{1}{11} \leq y \leq 1$

Maximum value = 1

16. $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$x^{a^2/bc}.x^{b^2/ca}.x^{c^2/ab} = x^{\frac{a^3+b^3+c^3}{abc}} = x^3$

17. $\cos\theta - 4\sin\theta = 1 - - - - - - (1)$

$\sin\theta + 4\cos\theta = 4 - - - - - - (2)$

$(1)^2 + (2)^2 \Rightarrow \cos^2\theta + 16\sin^2\theta + 16\cos^2\theta = 1 + x^2$

$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

18. $A+B+C=0 \Rightarrow A+B=-C \Rightarrow Cot(A+B) = -CotC$

$\sum \cot A \cot B = 1$

19. $\text{Sin}^{10} x - \cos^{10} x = 1 \Rightarrow \sin^{10} x = 1 + \cos^{10} x\ \ But\ \text{Sin}^{10} x \leq 1\ and\ 1 - \cos^{10} x \leq 1$

$\cos^{10} x = 0 \Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

20. $\text{Tan}\left[\frac{1}{2}\cos^{-1} 0\right] = \text{Tan}\left[\frac{1}{2}.\frac{\pi}{2}\right] \Rightarrow \text{Tan}\frac{\pi}{4} = 1$

21. $S=\frac{a+b+c}{2}=\frac{4+7+9}{2}=10\Rightarrow \text{Tan}\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{1}{\sqrt{20}}$

22. $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\left(\frac{r_1-r}{rr_1}\right)\left(\frac{r_2-r}{rr_2}\right)\left(\frac{r_3-r}{rr_3}\right)=\frac{4Rr^2}{r^2\Delta^2}=\frac{abc}{\Delta^3}$

23. circum radius of Δ, circum radius of pedal Δ

$=R:\frac{R}{2};1:\frac{1}{2};2:1$

24. $\sinh(ix)=\frac{e^{ix}-e^{-ix}}{2}=i\sin x$

25. Range of $\sin^{-1}5x=\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

26. $G.M=\left(4\times5\times20\times25\right)^{1/4}=(10000)^{1/4}=10$

27. $\rho=1-\frac{6\sum d^2}{n(n^2-1)}=1-\frac{6\times20}{5(5^2-1)}=1-1=0$

28. $f(x+y)=f(xy)$

If y=0 then f(x)=f(0) which is constant since f(2023)=2023

Now f(x)= 2023

29. $(fog)(x)=f\left[g(x)\right]=\log\left[\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right]=\log\left(\frac{1+x}{1-x}\right)^3=3f(x)$

30. $\left|\lambda(2\bar{i}-4\bar{j}+4\bar{k}\right|=1\Rightarrow|\lambda|=\frac{1}{6}\Rightarrow\lambda=\pm1/6$

31. ABCD is a parallelogram $\Rightarrow\overline{AB}=\overline{DC},\overline{BC}=\overline{AD}$

32. $\left(\bar{i}+3\bar{j}+4\bar{k}\right).\left(\lambda\bar{i}-4\bar{j}+\bar{k}\right)=0$, $\lambda=8$

33. $Radius=\frac{1}{2}\left|\left(2\bar{i}-3\bar{j}+\bar{k}\right)-\left(-3\bar{i}+\bar{j}-2\bar{k}\right)\right|=\frac{1}{2}\sqrt{25+16+9}=\frac{5}{\sqrt{2}}$

34. $\left(\bar{a}\times\bar{i}\right)^2+\left(\bar{a}\times\bar{j}\right)^2+\left(\bar{a}\times\bar{k}\right)^2=2a^{-2}$

37. Req no of ways = $7_{C_4}=35way$

$x=\max number\ appering\ as\ the\ die$

39. Let $P(x=4)=P(x\le4).P(x\le3)=\left(\frac{4}{6}\right)^5-\left(\frac{3}{6}\right)^5=\frac{1023}{6^5}$

40. $\frac{x}{(x-1)^2(x-2)}=\frac{a}{x-1}+\frac{b}{\left(x-1\right)^2}+\frac{c}{x-2}$

41. A(2,3),B(-1,1) Let P=(x,y)

$\underline{|APB}=90^0\Rightarrow PA^2+PB^2=AB^2,\ \ 2(x^2+y^2-x-4y+1)=0\Rightarrow x^2+y^2-x-4y+1=0$

42. Line equation is $y = x\tan 120^0 - 3 \Rightarrow y = -\sqrt{3}x - 3 \Rightarrow \sqrt{3}x + y + 3 = 0$

43. $(4,3)$ *lies on* $\frac{x}{2} - \frac{y}{3} = 1$ *and* $\frac{x}{-2} + \frac{y}{1} = 1$,

$2 - 3 = -1; -2 + 1 = -1 \Rightarrow$ *sum of the intercepts* $= -1$

44. Equation of the perpendicular is $y - 1 = -\frac{2}{3}(x-4) \Rightarrow 2x + 3y - 11 = 0$

Required ratio $= -\left[2(2)+3(-1)-11\right]:\left[2(6)+3(5)-11\right] = 10:16 = 5:8$

45.

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \Rightarrow 30k + \frac{1001}{4} - \frac{507}{4} - \frac{490}{4} - \frac{121k}{4} = 0 \Rightarrow -\frac{k}{4} + 1 = 0 \Rightarrow k = 4$$

46. Points of intersections of given pair of lines are same

47. Fourth vertex $= (4-2+3, 5-4+6, 1+1-3) = (5,7,-1)$

48. D.r's AB are $(-2-1, 4+2, 2-3) = (-3,6,-1) = (3,-6,1)$

D.C's of AB are $\left(\frac{3}{\sqrt{46}}, -\frac{6}{\sqrt{46}}, \frac{1}{\sqrt{46}}\right)$

49. $P = (2,4,-3), O = (0,0,0)$

D.r's of OP are(2,4,-3),D.c's of OP are $\left(-\frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}}, -\frac{3}{\sqrt{29}}\right), P = op = \sqrt{29}$

$\Rightarrow 2x + 4y - 3z = 29$

50. $$\underset{x\to 0}{Lt}\frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} = \underset{x\to 0}{Lt}\frac{\cos 2x - 1}{\sqrt{x^2+1}-1} = \underset{x\to 0}{Lt}\frac{-(1-\cos 2x)/x^2}{\left(\sqrt{1+x^2+1}-1\right)/x^2} = \frac{-2^2/2}{1/2} = -4$$

51. $$\underset{n\to\infty}{Lt}\frac{5.2^{n+1} + 2.3^{n+1}}{3.2^n - 7.3^n} = \underset{n\to\infty}{Lt}\frac{10(2/3)^n + 6}{3(2/3)^n - 7} = \frac{6}{-7}$$

52. For f to be continuous at x=2

$$\underset{x\to 0}{Lt}\frac{\log(1+(x^2+x^4))}{\sin^2 x}.\cos x = \underset{x\to 0}{Lt}\frac{\log(1+(x^2+x^4))}{x^2+x^4} \times \frac{x^2}{\sin^2 x} \times (1+x^2)\cos x = 1$$

53. $$\frac{dy}{dx} = \frac{1}{x\log a} - \frac{\log a}{x(\log x)^2}$$

54. $$\frac{d}{dx}\left[\frac{1}{4}(\log(1+x) - \frac{1}{4}\log(1-x) - \frac{1}{2}\mathrm{Tan}^{-1} x\right]$$

55. Find dx/dt and dy/dt then find dy/dx

56. $\frac{dy}{dx} = \frac{-(4x-y)}{-x+6y}$, *at* $(3,1)\ \frac{dy}{dx} = \frac{-11}{3}$

Find equation of Tangent

57. Find point intersection of two curves. Then find $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

58. $F(x)=\frac{x^2}{x+2}\Rightarrow f^1(x)=\frac{(x+2)2x-x^2}{(x+2)^2}=x(x+4)>0$

59. Find dy/dx *then for* max $dy/dx=0$ *we get* $x=e$

60. The line x+y=1 cuts x axis A(1,0) and y axis at B(0,1)

The equation of the circle having $\overline{AB}$ as diameter is (x-1)(x-0)+(y-0)(y-1)=0

$x^2+y^2-x-y=0$

61. Length of the tangent= $\sqrt{S_{11}}=\sqrt{1+9-2+12-11}=3$

62. Let r be radius of the circle

$d = distance\ from\ centre\ to\ line=\sqrt{29}$

Length of chord= 6

$\Rightarrow 2\sqrt{r^2-d^2}=6\Rightarrow r^2=38$

Equation circle $(x-3)^2+(y+1)^2=38$

63. Pole $=\left(-g+\frac{lr^2}{\lg+mf-n},-f+\frac{lr^2}{\lg+mf-n}\right)$

64. Given point A=(1,2) ,centre of the circle . C=(2,3)

ACB are collinear. Inverse point= (0,1)

65. $x^2+y^2+2x-2y+4=0, x^2+y^2+4x-2fy+2=0$

cuts orthogonally $2(1)(2)+2(-1)(-f)=4+2\Rightarrow 2f=2\Rightarrow f=1$

66. $\cos\theta=\frac{d^2-r_1^2-r_2^2}{2r_1r_2}$

67. given parabola is $y^2-4y-8x-4=0\Rightarrow y^2-4y+4=8x+8\Rightarrow (y-2)^2=8(x+1)$

Focus =(-1+2,2)=(1,2)

68. $y=2x+k\Rightarrow 2x-y+k=0, Here\ l=2, m=-1, n=k\ and\ a=1$

$\Rightarrow al^3+2alm^2+m^2n=0\Rightarrow k=-12$

69. $36x^2+144y^2-36x-96y-119=0\Rightarrow 36(x-1/2)^2+144(y-1/3)^2=144$

$a^2=4, b^2=1$

70. $10-a>0\Rightarrow a<10, 4-a>0\Rightarrow a<4\Rightarrow a<4$

71 Equation of pair of asymptotes is

$6x^2+13xy+6y^2-7x-8y+k=0, apply\Delta=0, we\ get\ k=2$

Equation asymptotes are $2x+3y-1=0; 3x+2y-2=0$

72. $\int\frac{x^4+x^2+1}{x^2+1}dx=\int\frac{x^2(x^2+1)+1}{x^2+1}dx=\frac{x^3}{3}+\mathrm{Tan}^{-1}x+c$

73. $I=\int\frac{x^4+1}{1+x^6}dx=\int\frac{(x^4-x^2+1)+x^2}{(1+x^6)}dx=\int\frac{x^4-x^2+1}{1+x^6}dx+\int\frac{x^2}{1+x^6}dx$

74. Let $\frac{2x+3}{(2x+1)(1-3x)}=\frac{A}{2x+1}+\frac{B}{1-3x}\Rightarrow 2x+3=A(1-3x)+B(2x+1)\Rightarrow A=4/5, B=11/5$

75. $4x+3=t^2\Rightarrow 4dx=2tdt\Rightarrow dx=t/2$

$$2x+3=\frac{1}{2}(4x+6)=\frac{1}{2}(t^2+3)$$

76. Let $I=\int_0^{\pi/2}\frac{\sec x}{\sec x+\cos ecx}dx=\int_0^{\pi/2}\frac{\sec(\pi/2-x)}{\sec(\pi/2-x)+\cos ec(\pi/2-x)}dx=\int_0^{\pi/2}\frac{\cos ecx}{\cos ecx+\sec x}dx$

$2I=\int_0^{\pi/2}\frac{\sec x+\cos ecx}{\sec x+\cos ecx}dx$; $2I=\int_0^{\pi/2}dx=[x]_0^{\pi/2}=\frac{\pi}{2}$

77. Let $I=\int_2^3\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx=\int_2^3\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$

$2I=1\Rightarrow I=1/2$

78. Curves are $y=x^2, y=2x$

$x^2-2x=0\Rightarrow x^2-2x=0\Rightarrow x(x-2)=0\Rightarrow x=0,2$

$x=0,2\Rightarrow y=0,4$, The two curves intersect at (0,0) and (2,4)

The area enclosed between the curves.

$\int_0^2(2x-x^2)dx$

79. Given equations is $\frac{dy}{dx}=e^{2x-y}+x^3e^{-3}\Rightarrow\frac{dy}{dx}=\frac{e^{2x}}{e^y}+\frac{x^3}{e^y}\Rightarrow e^{dy}=(e^{2x}+x^3)dx$

$e^y=\frac{e^{2x}}{2}+\frac{x^4}{4}+\frac{c}{4}\Rightarrow 4e^y=2e^{2x}+x^4+c$

The solution is $e^y=\frac{e^{2x}}{2}+\frac{x^4}{4}+\frac{c}{4}\Rightarrow 4e^y=2e^{2x}+x^4+c$, where c is an arbitrary constant

80. Put $y=vx, then\ \frac{dy}{dx}=v+x\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{y+x\cos^2(y/x)}{x}$

81. $\sin\left(\frac{5\pi}{3}\right)+\sec\left(\frac{13\pi}{3}\right)=\sin\left(2\pi-\frac{\pi}{3}\right)+\sec\left(4\pi+\frac{\pi}{3}\right)=-\sin\frac{\pi}{4}+\sec\frac{\pi}{3}=\frac{-\sqrt{3}}{2}+2$

82. $\tan\theta+\cot\theta=2\cos ec2\theta\Rightarrow\left(\tan 9^0+\cot 9^0\right)-\left(\tan 27^0+\cot 27^0\right)=$

$2\cos ec18^0-2\cos ec54^0$

$$=2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right]=4$$

83. $\cos 66^0+\sin 84^0=\cos 66^0+\cos 6^0=2\cos 36^0\cos 30^0=2\left(\frac{\sqrt{5}+1}{4}\right)\frac{\sqrt{3}}{2}=\sqrt{3}\frac{\left(\sqrt{5}+1\right)}{4}$

84. $\dfrac{{}^{2n}C_3}{{}^{n}C_3}=\dfrac{12}{1}\Rightarrow \dfrac{\dfrac{2n!}{(2n-3)!3!}}{\dfrac{n!}{(n-3)!3!}}=\dfrac{12}{1}$

$\dfrac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)}=12\Rightarrow \dfrac{2\times 2(2n-1)}{n-2}=12\Rightarrow 2n-1=3n-6$

$5=n$

85. $\tanh^2\left(\dfrac{\theta}{2}\right)=\dfrac{\cosh\theta-1}{\cosh\theta+1}=\dfrac{\sec x-1}{\sec x+1}=\dfrac{1-\cos x}{1+\cos x}=\dfrac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\dfrac{x}{2}$

86. $\sec^2 x+5\tan x+5$

$=1+\tan^2 x+5\tan x+5$

$=\tan^2 x+5\tan x+6$

$=\tan^2 x+2\tan x+3\tan x+6$

$=\tan x(\tan x+2)+3(\tan x+2)$

$=(\tan x+2)(\tan x+3)$

87. $c^2=a^2+b^2-2ab\cos c\Rightarrow c^2=a^2+b^2-ab$

Now $\dfrac{a}{b+c}+\dfrac{b}{c+a}=\dfrac{ac+a^2+b^2+bc}{bc+ab+c^2+ac}=1$

88. $\dfrac{1}{r}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}\Rightarrow \dfrac{1}{r}=\dfrac{1}{36}+\dfrac{1}{18}+\dfrac{1}{12}=\dfrac{2+4+6}{72}=\dfrac{1}{6}$

$r=6,\ \Delta^2=r.r_1.r_2.r_3=6^3\times 6^3\Rightarrow \Delta=216$

89. Give $f(10\times 3)=\dfrac{f(10)}{3}=20\Rightarrow f(10)=60$

$f(40)=f(10\times 4)=\dfrac{f(10)}{4}=\dfrac{60}{4}=15$

90. Let $y=f(x)=\dfrac{x^2+x+1}{x}$

$xy=x^2+x+1\Rightarrow x^2x(1-y)+1=0$

$\Delta\geq 0\Rightarrow b^2-4ac\geq 0\Rightarrow (1-y^2)-4\geq 0$

$1+y^2-2y-4\geq 0\Rightarrow y^2-2y-3\geq 0$

$y(y-3)+1(y-3)\geq 0$

$(y+1)(y-3)\geq 0$

$y\in(-\propto,-1]\cup[3,\propto)$

91. $A=\begin{bmatrix}\alpha^2 & 5\\ 5 & -\alpha\end{bmatrix}$ and $|A^{10}|=1024$

$|A| = -\alpha - 25 \quad |A^{10}| = 2^{10} \qquad \alpha^3 - 25 = -2$

$|A| = \pm 2 \qquad -\alpha^3 = 23 \quad , \quad \alpha^3 = -23$

$-\alpha^3 - 25 = 2 \Rightarrow -\alpha^3 = 27 \Rightarrow \alpha \; = -27 \qquad ga = (-23)^{\frac{1}{3}}$

$\alpha = -3$

92.

$$A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ -1 & 2 & 3 & 5 \\ 0 & 1 & k & k \end{bmatrix} . R_2 \to R_2 + R_1 \Rightarrow \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 4 & 4 & 4 \\ 0 & 1 & k & k \end{bmatrix} R_3 \to 4R_3 - R_2 \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & k-1 & k-1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

Rank of matrix A will be 2 if k-1 = 0 $\Rightarrow k = 1$

Satisfy the equation $x^2 + x - 2 = 0$

93. $\begin{bmatrix} 5 & a & -7 \\ b & -7 & c \\ -7 & d & -1 \end{bmatrix}$ is adjoint of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} =$

$$\begin{bmatrix} 5 & a & -7 \\ b & -7 & c \\ -7 & d & -1 \end{bmatrix} = \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$$

$a = -1, \; b = -1, \; c = 5, \; d = 5$

94. Let $\overline{OA} = \bar{a}, \; \overline{OB} = \bar{b}, \; \overline{OC} = \bar{c}, \; \overline{OD} = \bar{d}$

$$\overline{AM} + \overline{AN} = (\overline{OM} - \overline{OA}) + (\overline{ON} - \overline{OA}) = \left(\frac{\bar{b}+\bar{c}}{2} - \bar{a}\right) + \left(\frac{\bar{c}+\bar{d}}{2} - \bar{a}\right)$$

$$= \frac{\bar{b} + \bar{d} + 2\bar{c} - 4\bar{a}}{2}$$

$$= \frac{\bar{c} + \bar{a} + 2\bar{c} - 4\bar{a}}{2} \qquad \left[\because \bar{a} + \bar{c} = \bar{b} + \bar{d}\right]$$

$$= \frac{3(\bar{c} - \bar{a})}{2} = \frac{3}{2}\overline{AC}$$

95. $\bar{a} + \bar{b} + \bar{c} = 0 \Rightarrow (\bar{a} + \bar{b} + \bar{c})^2 = 0 \Rightarrow |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}) = 0$

$\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a} = -\frac{1}{2}(1 + 9 + 16) = -13$

96. $|\bar{a} \times \bar{b}|^2 = |\bar{a}|^2 |\bar{b}|^2 - (\bar{a}.\bar{b})^2 = 169 \times 25 - (60)^2 = 4225 - 3600$

$|\bar{a} \times \bar{b}|^2 = 625 \Rightarrow |\bar{a} \times \bar{b}|^2 = 25$

97. If $\bar{a}$ and $\bar{b}$ are collinear then $\bar{b} = \lambda\bar{a}$

$\left|\bar{b}\right| = |\lambda|\left|\bar{a}\right| \Rightarrow 21 = |\lambda|\sqrt{4+9+36} \Rightarrow |\lambda| = \frac{21}{7} = 3$

$\lambda = \pm 3$

$\therefore \bar{b} = \pm 3\left(2\hat{i} + 3j + 6k\right) = \pm\left(6\hat{i} + 9j + 18k\right)$

98. Let $\bar{b} = x\bar{i} + y\bar{j} + z\bar{k}$, if $\bar{a}\times\bar{b} = \bar{c}$ then $\bar{c}$ $\perp$ lar to $\bar{a}$ and $\bar{b}$

$\bar{b}.\bar{c} = 0 \quad y - z = 0 \Rightarrow y = z$

$\bar{a}.\bar{b} = 3 \quad x + y + z = 3 \Rightarrow x + 2y = 3$

$\bar{a}\times\bar{b} \Rightarrow \begin{vmatrix} \hat{i} & j & k \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = j - k \Rightarrow \hat{i}(z-y) - j(z-x) + k(y-z) = j - k$

$x - z = 1$ and $x - y = 1$ $\qquad x = \frac{5}{3},\ y = \frac{2}{3}$

$\bar{b} = \frac{1}{3}\left(5\hat{i} + 2j + 2k\right)$

99. $\left(\lambda\hat{i} - 3j + 5k\right).\left(2\lambda\hat{i} - \lambda j + k\right) = 0$

$2\lambda^2 + 3\lambda + 5 = 0$

As D = 9-40<0

no real value of λ satisfy the equation

100. $x + 1 = A\, x + 1 = A(3x+1) + b(2x-1)$------(i)

Put $x = \frac{1}{2}$ and $-\frac{1}{3}$ in (i) we get $A = \frac{3}{5}$ and $B = -\frac{2}{5}$

$16A + 9B = 16\left(\frac{3}{5}\right) + 9\left(\frac{-2}{5}\right) = \frac{48-18}{5} = \frac{30}{5} = 6$

101. put $3^x = y$ $\qquad 3.y + \frac{3}{y} = 10 \Rightarrow 3y^2 - 10y + 3 = 0 \Rightarrow y = 3, \frac{1}{3}$

$3^x = 3^1 \qquad 3^x = 3^{-1}$

No of +ve real roots is 1

102. $\Delta = b^2 - 4ac = 0 \Rightarrow (2m+1)^2 - 4m = 0 \Rightarrow 4m^2 + 4m + 1 - 4m = 0$

$m^2 = -\frac{1}{4}$ (which is not possible)

m ras no real values

103. let α be the common roots of $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$

$\alpha^3 + a\alpha + 1 = 0$ --(1) $\quad \alpha^4 + a\alpha^2 + 1 = 0$ --(2)

(1) $\quad \alpha^4 + a\alpha^2 + \alpha = 0$ --(3)

From (1) $\quad 1 + a + 1 = 0 \Rightarrow a = -2$

104. $\alpha = 1,\ \beta = 2 + \sqrt{3},\ r = 2 - \sqrt{3}$

$(x-\alpha)(x-\beta)(x-\gamma) = x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$

$x^3-5x^2+3x+1=0$

105. $\frac{1+i}{1-i}=i \quad \frac{1-i}{1+i}=-i$

$(-i)^{2022}+(i)^{2021}=(-i)^2+(i)^1 \qquad i^4=1 \Rightarrow (i^4)^{505}=1$

$=i^2+i=-1+i=i-1$

106. If z_1, z_2, z_3 are vertices of an equilateral triangle then $z_1^2+z_2^2+z_2^3=z_1z_2+z_2z_3+z_3z_1$

In this equation $z_3=0$

$z_1^2+z_2^2=z_1z_2$

107. $1+\omega+\omega^2=0 \Rightarrow 1+\omega^2=-\omega$, $1+\omega=-\omega^2$

$=(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5=(-2\omega)^5+(-2\omega^2)^5$

$=-32(\omega^5+\omega^{10})=-32(\omega^2+\omega)=-32(-1)=32 \qquad (\because \omega^3=1)$

108. middle term is $\frac{n}{2}+1=\frac{20}{2}+1=11$

$m=11$

$T_{m+3}=T_{11+3}=T_{13+1}={}^{20}C_{13}(x^2)^{20-13}\left(-\frac{1}{2x}\right)^{13}=\frac{{}^{20}C_{13}x^7(-1)^{13}}{2^{13}x^{13}}$

So coefficient is ${}^{20}C_{13}.2^{-13}$

109. given $x=\frac{5}{2!3}+\frac{5.7}{3!3^2}+\frac{5.7.9}{4!3^3}$

$x+1+\frac{3}{1!}\left(\frac{1}{3}\right)=1+\frac{3}{1!}\left(\frac{1}{3}\right)+\frac{3.5}{2!}\left(\frac{1}{3}\right)^2+........$

$2+x=\left(1-\frac{2}{3}\right)^{\frac{-3}{2}} \Rightarrow 2+x=3^{\frac{3}{2}}$

$(2+x)^2=\left(3^{\frac{3}{2}}\right)^2 \Rightarrow x^2+4x+4=27$

$x^2+4x=23$

110. Let number of sides of the polygon is n, then umber of diagonals is

$\frac{n(n-3)}{2}=54 \Rightarrow n(n-3)=108$

$n=12$

111. Number of ways to arrange 6 red balls are 6!

Now, there are seven positions to place 6 black balls, such that no two black balls are together

So the number of ways to arrange 6 black balls are ${}^7P_6$

The required number of arrangements $=6!\times{}^7P_6=6!\times 7!$

112. A number divisible of 4 formed by the digits 1, 2, 3, 4, 5 should have the last two digits 12 (or) 32 (or) 52

Remaining 3 digits in 3! = 6 ways

A number divisible by 4can be formed in $=6\times4=24$ ways

Required probability = $\frac{24}{120}=\frac{1}{5}$

113. total number of balls $7+5=12$

While drawing, ball is not replaced

Required probability $=\frac{{}^{7}C_{1}.\ {}^{6}C_{1}.\ {}^{5}C_{1}}{{}^{12}C_{1}.{}^{11}C_{1}.{}^{10}C_{1}}=\frac{7.6.5}{12.11.10}=\frac{7}{44}$

114. total number of out comes when 3 cards are drawn ${}^{52}C_{3}$

Prob (ace, jack and queen) $=\frac{{}^{4}C_{1}\times{}^{4}C_{1}\times{}^{4}C_{1}}{{}^{52}C_{3}}=\frac{4\times4\times4}{\frac{52\times51\times50}{1\times2\times3}}=\frac{16}{5525}$

115. Given , mean of $B(n,p)=np=15$

Variance of $B(n,p)=npq=10$

$np=15$ and $npq=10$

$np(1-p)=10$

$15(1-p)=10$

$1-p=\frac{10}{15}$

$1-p=\frac{2}{3}\Rightarrow 1-\frac{2}{3}=p$

$p=\frac{1}{3}$

116. mean = variance = 4

So $e^{4}\left[1-p(x>20)\right]=e^{4}\left[p\le 2\right]$

$=e^{4}\left[p(x=0)+p(x=1)+p(x=2)\right]$

$=e^{4}\left[\frac{e^{-4}\times4^{0}}{0!}+\frac{e^{-4}4!}{1!}+\frac{e^{-4}\times4^{2}}{2!}\right]$

$=e^{4}e^{-4}(1+4+8)=13$

117. For 60 observations it's given that $\sum x_i=960$ and $\sum x_i^2=18000$

So the variance $\frac{\sum x_i^2 60}{60}-\left(\frac{\sum x_i}{60}\right)^2$

$$\frac{1800}{60}-\left(\frac{960}{60}\right)^2=300-256=44$$

118. $\bar{x}=\frac{\sum x_i}{n}=\frac{3+5+11+13+17+19+29}{8}=15$

Mean deviation about mean

$$=\frac{|3-15|+|5-15|+|11-15|+|13-15|+|17-15|+|19-15|+|23+15|+|29-15|}{8}$$

$$\frac{56}{8}=7$$

119. a is not invertible matrix $\Rightarrow |A|=0$

$$\begin{vmatrix} 1 & 1 & a+1 \\ 1 & a+1 & 1 \\ a+1 & 1 & 1 \end{vmatrix}=0 \Rightarrow 1(a+1-1)-(1-a-1)+(a+1)\left(1-(a+1)^2\right)=0$$

$$-a^3-3a^2=0 \Rightarrow -a^2(a+3)=0 \Rightarrow a=0,-3$$

Sum of all values $=0-3=-3$

120. $$\frac{i^8\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)^8}{(-i)^8\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)^8}=\frac{cis\left(8\left(-\frac{\pi}{8}\right)\right)}{cis\left(8\left(\frac{\pi}{8}\right)\right)}=\frac{cis(-\pi)}{cis(\pi)}=cis(-2\pi)$$

$$=\cos 2\pi - i\sin 2\pi$$
$$=1-0=1$$

121. $$(x,y)=\left(\frac{p+a\cos\theta+b\sin\theta}{3},\frac{q+b\sin\theta+a\cos\theta}{3}\right)$$

$$3x-p=(a+b)\cos\theta;\ 3y-q=(a+b)\sin\theta$$

Eliminating 'θ' we get

$$(3x-p)+(3y-q)^2=(a+b)^2$$

122. $$PA+PB+AB=14$$
$$PA+PB=8$$
$$pA=8-PB \Rightarrow PA^2=64+PB^2-16PB$$
$$(x+3)^2+y^2=64+(x-3)^2+y^2-16PB$$
$$12x=64-16PB$$

S.O.B s we get $7x^2+16y^2=112$

123. The distance between $\left(\frac{2}{m},2\right)$ and $\left(\frac{6}{m},6\right)$ is less than 5

Hence $\left(\frac{6}{m}-\frac{2}{m}\right)^2+(2-61)^2<25$

$$m^2-\frac{16}{9}>0$$

$$\left(m+\frac{4}{3}\right)\left(m-\frac{4}{3}\right)>0$$

124. Any point on the line $2x-y=0$ is $B(\alpha,2\alpha)$

Any point on the line $x+y=3$ is $(\beta,3-\beta)$ $A(1,2)$

$2=\frac{1+\alpha+\beta}{3}$, $3=\frac{2+2\alpha+3-\beta}{3}$

$\alpha+\beta=5,\ 2\alpha-\beta=4 \Rightarrow \alpha=3, \beta=2$

$B=(3,6), C=(2,1)$

Equation of BC is $5x-y-9=0$

125. $S=\left(\frac{7+6}{2},\frac{3-1}{2}\right)=\left(\frac{13}{2},1\right)$

Slope of PS= $\frac{2-1}{2-\frac{13}{2}}=\frac{-2}{9}$

126. $y^2\left(x^2-2x-3\right)-4y\left(x^2-2x-3\right)=0$

$\left(y^2-4y\right)\left(x^2-2x-3\right)=0$

$y^2-4y=0 \Rightarrow y=0,\ y=4$

$x^2-2x-3=0 \Rightarrow x=-1, x=3$

127. Let $T_r=\frac{1}{(r+2)r!}=\frac{(r+1)}{(r+2)!}=\frac{(r+2)-1}{(r+2)}$

$=\frac{1}{(r+1)!}-\frac{1}{(r+2)!}$

$a=\underset{n\to\infty}{lt}\sum_{r=1}^{n}\left[\frac{1}{(r+1)!}-\frac{1}{(r+2)!}\right]$

$a=\underset{n\to\infty}{lt}\left[\frac{1}{2}-\frac{1}{(n+2)!}\right]=\frac{1}{2}$

And $b=\underset{n\to\infty}{lt}\frac{e^x\left(e^{\sin x-x}-1\right)}{\sin x-x}=1$

128. L= $\underset{x\to\frac{\pi}{4}}{Lt}\frac{4\sqrt{2}\left[1-\cos^5\left(x-\frac{\pi}{4}\right)\right]}{1-\cos 2\left(x-\frac{\pi}{4}\right)}$

$=\underset{y\to 0}{Lt}\frac{4\sqrt{2}\left[1-\cos^5 y\right]}{1-\cos 2y}$

$u\sin g$ L-Hospital rule

$=5\sqrt{2}$

129. $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two lies $px+qy+r=0$

$lx+my+n=0$ where $a=lp, 2h=lq+mp$,

$b=ma, c=nr$

$ax^2+2hxy+by^2-2gx-2fy+c=0$ represents two lines

$px+qy-r=0$

$lx+my-n=0$

Area of parallelogram $=\left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_2}\right|$

$$=\frac{2|c|}{\sqrt{h^2-ab}}$$

130. $f(x)$ is continuous at $x=\sqrt{3}$

$$f\left(\sqrt{3}\right)=\lim_{x\to\sqrt{3}}\frac{x^2-2x+2\sqrt{3}-3}{\sqrt{3}-x}$$

131. $\log y=\sin x\log\left(x^2+1\right)$

$$\frac{1}{y}\frac{dy}{dx}=\cos x.\log\left(x^2+1\right)+\frac{\sin x}{x^2+1}2x$$

$$\frac{dy}{dx}=\left(x^2+1\right)^{\sin x}\left(\cos x.\log\left(x^2+1\right)+\frac{2x\sin x}{x^2+1}\right)$$

$y'(0)=0$

132. $g\left(f(x)\right)=x\Rightarrow g'\left(f(x)\right)f'(x)=1$

$$\Rightarrow g'\left(f(x)\right)=\frac{1}{1+\sec^2 x}=\frac{1}{2+\tan^2 x}$$

Replace x by $g(x)$

$$\Rightarrow g'(x)=\frac{1}{2+\left[g\left((x)-x\right)\right]^2}$$

133. $\frac{dx}{dy}=\frac{1}{\cos x+e^x}$

134. The parameter equation of the curve are $x=aa\cos^3\theta,\ y=a\sin^3\theta$

Eq. of tangent is $y-a\sin^3\theta=-\tan\theta\left(x-a\cos^2\theta\right)$

$$x\sin\theta+y\cos\theta=a\sin\theta\cos\theta$$

Eq. of normal is $y-a\sin^3\theta=+\cot\theta\left(x-a\cos^2\theta\right)$

$x\cos\theta-y\sin\theta=a\cos 2gq$

$p=a\sin\theta\cos\theta$ $\qquad q=a\cos 2\theta$

$4p^2+q^2=a^2$

135. $2y=e^{-x/2}$

$x=0\Rightarrow y=\frac{1}{2}$

$\frac{dy}{dx}=\frac{1}{4}e^{-x/2}$

$$m_1 = \left[\frac{dy}{dx}\right]_{\left(0,\frac{1}{2}\right)} = -\frac{1}{4}$$

Eq of tangent at $\left(0,\frac{1}{2}\right)$ is

$x+4y-2=0$

136. $f'(x)=3kx^2-18x+6$

$3\left(kx^2-6x+2\right)$

$\Delta<0 \Rightarrow 36-4.k.2<0$

$\Rightarrow k>\frac{9}{2}$

137. minimum value of $f(x)=\frac{1}{1+b^2}=m(b)$

Range of $m(b)=(0,1]$

138. $4+a^2+b^2=21 \Rightarrow a^2+b^2=17$

$-6+3a-6b=0 \Rightarrow 3a=6+6b$

$\Rightarrow a=2+2b$

$\Rightarrow 4+4b^2+8b=17$

$\Rightarrow 5b^2+8b-13=0 \Rightarrow b=1,\ a=4$

139. eq. of plane is $\begin{vmatrix} x-4 & y-4 & z=0 \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{vmatrix}=0$

$(x-4)(-2)-(y-4)(-4)+z(-3)=0$

$-2x+8+4y-16-3z=0$

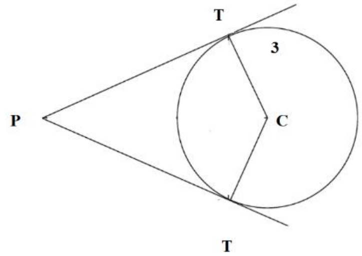
$2x-4y+3z+8=0$

140. $\cos 60^0 = \frac{CM}{CP}$

$\frac{1}{2}=\frac{\sqrt{(x_1-1)^2+(y_1-1)^2}}{2} \Rightarrow (x_1-1)^2+(y_1-1)^2=1$

$x^2+y^2-2x-2y+1=0$

141. $PT=\sqrt{16+16-8-8-7}=3$



$TT'=2BT=2.3.\cos 45^0$

$=3\sqrt{2}$

142. Circum center is mid point of AB

Eq. of line AB is $\frac{x}{a}+\frac{y}{b}=1$

$(h,k)=\left(\frac{a}{2},\frac{b}{2}\right)\Rightarrow a=2h_1,\ b=2k$

$\Rightarrow \frac{x}{2h}+\frac{y}{2k}=1$

$\Rightarrow kx+hy-2hk=0$

$r=d\Rightarrow \frac{|2h+2k-2hk|}{\sqrt{k^2+h^2}}=2$

$h+k-hk=-\sqrt{h^2+k^2}$

143. The triangle is right angled and the radical centre will be the artho-centre of the triangle

144. $\sqrt{d^2-(r_1-r_2)^2}$ where $d=c_1c_2$

145. (perpendicular distance form P to axis $)^2$

= L-L-R (Perpendicular distance form P to tange)

$\left(\frac{x+y-2}{\sqrt{1+1}}\right)^2=2\left(\frac{x-y+4}{\sqrt{2}}\right)$

146. $yt=x+at^2$

$P=(0,at)$, $T=(-at^2,0)$, $Q=(-at^2,at)=(x,-y)$

$t=\frac{y}{a}, x=-at^2\Rightarrow x=-\frac{y^2}{a}\Rightarrow y^2+ax=0$

147. Eq of normal to $y^2=4x$ on $P(t^2,2t)$ is

$y+xt=2t+t^3$ passes through $C(6,0)$

$t^2-4t=0\Rightarrow t=2\Rightarrow P(4,4)$

Minimum distance $=CP-r=2\sqrt{5}-\sqrt{5}=\sqrt{5}$

148. $\frac{x^2}{16}+\frac{y^2}{9}=1\ a=4,\ b=3,\ e=\frac{\sqrt{7}}{4}$

Least radius $=\frac{1}{2}$ (distance between the foci)

$ae=\sqrt{7}$

149. $s=xy-3y-2x=0$

$\frac{\partial s}{\partial x}=y-2=0\ \ \frac{\partial s}{\partial y}=x-3=0$

Center of the hyperbola $=(3,2)$

Eq. of asymptotes is $xy-3y-2x+k=0$

$6-6+k=0$

$k = 6$

150. Eq. of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Co ordinates of foci are $(\pm ae, 0) = (\pm 4, 0)$

Eq. of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Co ordinates of foci are $(\pm ae, 0) = (\pm 4, 0)$

$ae = 4 \Rightarrow a = 2$

$\Rightarrow b^2 = a^2(e^2 - 1) = 12$

$\frac{x^2}{4} - \frac{y^2}{12} = 1$

151. $\int \frac{x^2\left(\sqrt{1+x^2}+1\right)}{\sqrt{1+x^2}\left[\left(1+x^2\right)-1\right]} dx = \int \left(1 + \frac{1}{\sqrt{1+x^2}}\right) dx$

$= x + \sinh^{-1} x + c$

152. put $4x + 3 = t^2 \Rightarrow dx = t\frac{dt}{2}$

$I = \int \frac{(t^2 - 3)\sqrt{t^2}}{4} \frac{t}{2} dt = \frac{1}{8}\int \left(t^4 - 3t^2\right) dt$

$= \frac{1}{8}\left[\frac{t^5}{5} - \frac{3t^3}{3}\right] + c$

$= \frac{1}{40}\left[(4x+3)^{\frac{5}{2}} - 5(4x+3)^{\frac{3}{2}}\right] + C$

153. $I = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$

$\sqrt{2} = \int \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} dx$

$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + C$

154. $I = \int_{\frac{1}{3}}^{1} -\log_e^x dx + \int_{1}^{3} \log_e^x dx$

$= -\left[x \log_e^x - x\right]_{\frac{1}{3}}^{1} + \left[x \log_e^x - x\right]_{1}^{3}$

$= -\frac{4}{3} + \frac{8}{3}\log_e^3 = \frac{4}{3}\log^{\left(\frac{9}{e}\right)}$

155. put $x^2 = t \Rightarrow x dx = \frac{dt}{2}$

$I=\int \frac{1}{(t+1)(t+3)}=\frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)$

$f(x)=\frac{1}{2}\log\left(\frac{x^2+1}{x^2+3}\right)+c$

$f(3)=\frac{1}{2}\log\left(\frac{10}{12}\right)+c\Rightarrow c=0$

$f(4)=\frac{1}{2}\log_e^{\left(\frac{17}{19}\right)}$

156. $I=\int_0^1\left(x^{20}+x^{13}+x^6\right)\left(2x^{21}+3x^{14}+6x^2\right)^{\frac{1}{7}}dx$

Put $2x^{21}+3x^{14}+6x^7=t$

$=\frac{1}{42}\int_0^{11} t^{\frac{1}{7}dt}=\frac{1}{48}\left(t^{\frac{8}{7}}\right)_0^{11}=\frac{1}{48}(11)^{\frac{8}{7}}$

157. given $y=\sin 2x$

$y=\sqrt{3}\sin x$

For point of interjection of 1 & 2 we get

$x=0,\ x=\frac{\pi}{6}$

Are $=\int_0^{\frac{\pi}{6}}\left(\sin 2x-\sqrt{3}\sin x\right)dx$

$=\left(\frac{7}{4}-\sqrt{3}\right)$ sq. units

158. $\frac{dx}{dy}=\frac{e^{x/y}\left(\frac{x}{y}-1\right)}{\left(e^{x/y}+1\right)}\rightarrow(1)$

Put $\frac{x}{y}=v\Rightarrow\frac{dx}{dy}=x+y\frac{dx}{dy}$

1 becomes $v+y\frac{dx}{dy}=\frac{e^v(v-1)}{\left(e^v+1\right)}$

$y\frac{dv}{dy}=\frac{-\left(e^v+v\right)}{e^v+1}$

$\log\left(e^v+v\right)+\log y=\log c$

159. $x=e^{dy/dx}\Rightarrow\frac{dy}{dx}=\log_e^x$

Order =1 degree =1

160. $L=(2,4,0),\ M=(0,4,5)$

$LM=\sqrt{4+25}=\sqrt{29}$

161. $\dfrac{3\left(\dfrac{x-1}{x+1}\right)+1}{\left(\dfrac{x-1}{x+1}\right)+3}=\dfrac{3x-3+x+1}{x-1+3x+3}=+2x$

162. $(x-1)(3-x)=\left[x^2-4x+3\right]\Rightarrow-\left[x^2-4x+4-1\right]=1-(x+2)^2$

Range $[0,1]$

163. $A=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix},A^2=\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix},A^3=\begin{bmatrix}2&3&3\\3&2&3\\3&3&2\end{bmatrix},i.eA^3-A^2=2A$

164. $(x^2-4)^3=125\Rightarrow\alpha^2=9\Rightarrow\alpha=\pm3$

165. $a^3+b^3=0\Rightarrow\left(\dfrac{a}{b}\right)^3=-1\ or\ \left(\dfrac{b}{a}\right)^3=-1$

$\dfrac{b}{a}$ is one cube root of -1

166. $XA=B\Rightarrow X=BA^{-1},YB=A\Rightarrow Y=AB^{-1}$

$A^{-1}=\dfrac{1}{3}\begin{bmatrix}-1&1\\-4&1\end{bmatrix},B^{-1}=\dfrac{1}{3}\begin{bmatrix}1&1\\-2&1\end{bmatrix}$

$3(x+y)=\begin{bmatrix}3&0\\-6&3\end{bmatrix}+\begin{bmatrix}3&0\\6&3\end{bmatrix}=6$

167. x>0, $f(x)=\dfrac{x}{1+x},\ f^1(x)=\dfrac{1}{(1+x)^2}>0 one-one$

For onto, $y=\dfrac{x}{1+x}\Rightarrow x=\dfrac{y}{1-y},y\neq1\,not\ onto$

168. $f(x)=f^{-1}(x)\Rightarrow f(x)=x\Rightarrow(x+1)^2-1=x$

169. $S=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix},S^{-1}=\begin{bmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{bmatrix}$

$SA=2\begin{bmatrix}0&2a&2a\\2b&0&2b\\2c&2c&0\end{bmatrix},SAS^{-1}=\begin{bmatrix}4a&0&0\\0&4b&0\\0&0&-14c\end{bmatrix}$

Trace $\left[SAS^{-1}\right]=4(a+b+c)$

170. Let $T_{P+1}\ term\ is\ x^{2t};P=\dfrac{n-2r}{1+2}\Rightarrow n-2r=3P$

171. Let $e^{\sin x} = a, we\ get\ a^2+4a+1=0$, $a=-12\pm\sqrt{3}$

$e^{\sin x} = 2+\sqrt{3}\ or\ 2-\sqrt{3}$ not possible

No real solution

173. $ax^2+x+b=0$, $D_1 = 1-4ab \geq 0$

$ax^2-4\sqrt{ab}x+1=0$, $D_2 = 16ab-4\leq 0$ roots are imaginary

175. $\frac{x+1}{x^2-px+2} = \frac{A}{x-\alpha}+\frac{B}{x-\beta}$ and $v+b=P\ \alpha\beta=9$, $\frac{A-B}{A+B} = \frac{P+2}{\sqrt{P^2-49}}$

176. Word Triangle, Vowels $A\in 2,(3)$

Components TRNGL (5)

Angle 3!5!=720

177. $W\alpha = 2-3i\ \ and\ \ b=4; r=\sqrt{\alpha\overline{\alpha}b}=3$

178. $4+5\omega^{334}+3\omega^{365} \Rightarrow 4+5\omega+3\omega^2 = 1+2\omega = \sqrt{3}i$

179. Verify with option

180. Concept A+B=90 and 2 Tan(A-B)=Tan A –Tan B

181. $\cos(\alpha+\beta) = \frac{2\,\mathrm{Tan}\left(\frac{\alpha+\beta}{2}\right)}{1-\mathrm{Tan}^2\frac{\alpha+\beta}{2}}$

182. $\cos 2\alpha = \frac{3\cos^2\beta-1}{3-\cos^2\beta}$

By compodendo and dividend $\tan^2\alpha = 2\,\mathrm{Tan}^2\beta$

183. $6-15+10+7=8$

185.

$r_1+r_2 = 4R\sin A/2\cos B/2\cos C/2+4R\cos A/2\sin B/2\cos C/2 \Rightarrow 4R\cos^2 C/2 = 2R(1+\cos x)$

186. $c^2 = a^2+b^2-2bc\cos c \Rightarrow c=\sqrt{6}$

187. $\cos A+\cos B+\cos C = \frac{7}{4}$

$1+\frac{r}{R} = \frac{7}{4} \Rightarrow \frac{r}{R} = \frac{3}{4}$

188. $\left|\overline{a}+\overline{b}\right| < \left|\overline{a}-\overline{b}\right| \Rightarrow \cos\theta<0, \theta\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

189. $\overline{\alpha} = \overline{a}+\overline{b}+\overline{c} \Rightarrow bi+12j = 2\overline{a}-3\overline{b}$

190. Equation plane OAC is $\begin{vmatrix} x & y & z \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0 \Rightarrow 3x+y-5z=0$

Shortest distance of B(1,2,1) from plane $d = \left|\frac{3\times1+2\times1-5\times1}{\sqrt{3^2+1^2+(-5)^2}}\right| = 0$

191. $\frac{[a\ b\ c]}{[a\ b\ c]}+\frac{[b\ c\ a]}{[c\ a\ b]}+\frac{[c\ b\ a]}{[a\ b\ c]}=1+1-1=1$

193. $\left(\bar{a}.\bar{c}\right)\bar{b}-\left(\bar{a}.\bar{b}\right)\bar{c}=\frac{\bar{b}+\bar{c}}{\sqrt{2}}$

$\left(\bar{a}.\bar{c}\right)=\frac{1}{\sqrt{2}}\qquad \bar{a}.\bar{b}=\frac{-1}{\sqrt{2}}$

$\bar{a}\ \bar{b}\ \bar{c}$ are non coplanarcenet vector

$\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$

194. $2(a)(b)\sin\theta+3(a)(b)\cos\theta$

max $\Rightarrow\sqrt{4\times4\times9+9\times4\times9}=2\times3\sqrt{4+9}=6\sqrt{13}$

195. $\frac{(x-20)(x-40)}{x-30}<0\qquad probability\ =\frac{28}{100}=\frac{7}{25}$

$1,2,3,4,5,.....19,31,32,.....39$

196. $P(B)=x\quad P(A)+P(B)=P(A\cup B)+P(A\cap B)$

$$2x+x=\frac{7}{8}\Rightarrow x=\frac{7}{24},P(A)=2x=\frac{7}{12}$$

197. $P(s)=\frac{2}{3}-P;P(F)=\frac{1}{3}q$

201. Let (x,y) be point

$(x,y)(a,b)(b,a)$ are collinear

$\frac{x-a}{x-b}=\frac{y-b}{b-a}\Rightarrow x-a=-y+b,x+y=a+b$

202. diagonal are bisects to each other

203. $\frac{d_1}{d_2}=\frac{|c_1-c_2|}{|c_2-c_3|}=\frac{(5/3/2)}{\left|\frac{3}{2}+5\right|}=7/13$

204. As c_1,c_2 having same sign. Bisector is angle having origion

$\frac{d_1}{d_2}=\frac{|4x+3y-7|}{\left|\sqrt{3^2+4^2}\right|}=+\left|\frac{24x+7y-31}{\sqrt{24^2+7^2}}\right|\Rightarrow5(4x+3y-7)=24x+7y-31\Rightarrow x-2y+1=0$

205. $OR=\left|\frac{4(0)+3(0)+30}{\sqrt{4^2+3^2}}\right|,PQ=2\sqrt{10^2-6^2}=2\times8=16$

206. Seem of reciprocal of intercepts

$\frac{1}{a}+\frac{1}{b}=\frac{1}{2}\Rightarrow\frac{2}{a}+\frac{2}{b}=1\Rightarrow(2,2)\to\frac{x}{a}+\frac{y}{b}=1$

207. $(x^2+y^2)\tan\alpha=(x-y\tan\alpha)^2$

$$\tan\alpha(x^2+y^2)=x^2-2xy\tan\alpha+\tan^2\alpha$$

$$2\tan\alpha=1+\tan^2\alpha\Rightarrow\tan^2\alpha-2\tan\alpha+1=0\Rightarrow\tan\alpha=1\Rightarrow\frac{\pi}{4}$$

208. $x^2+y^2=a^2+b^2.........circle$

209. AB+ 6 cm
R= 5cm and diameter = 10 cm
No. of circle=2

210. Centre $\left(-\alpha,\alpha\right)$, $x^2+y^2+2\alpha x+2\alpha y+\alpha^2=0\Rightarrow 3x-4y+8=0$

211. As $g^2=c\ touches\ x-axis$

212. Centre $(a\cos\alpha\ a\sin\alpha), radius\Rightarrow x-axis, \left|a\cos^2\alpha+a\sin^2\alpha\right|=a$

$a-p=\pm a\Rightarrow p=0\ or\ 2a$

213. $x^2+y^2+2gx+2fy+c=0$
$(1,2)\to 2g+4f+c=-5........(1)$
$(3,4)\to 6g+8f+c=-25.....(2)$
By solving it $g==-4,\frac{-3}{2}\ f=-1,\frac{-7}{2}$

214. $d=c_1c_2=5\Rightarrow R-r>5$

215. $(x+a)^2+(y+b)^2=a^2\Rightarrow r_1=a,c_1=(-a,-b)$
$(x+\alpha)^2+(y+\beta)^2=\beta^2\Rightarrow r_2=\beta,c_2=(-\alpha,-\beta)$ circles touches orthogonally
$r_1^{\ 2}+r_2^{\ 2}=(c_1c_2)^2\Rightarrow\alpha^2+b^2=2(a\alpha+b\beta)$

216. $\frac{x-3}{\cos 30}=\frac{y-4}{\sin 0}=r$

$x=\frac{3+\sqrt{3}r}{2}, y=4\pm\frac{5}{2}\Rightarrow 12x+5y+10=0\Rightarrow r=\frac{132}{12\sqrt{3}+5}$

217. $x^2-4\sin x-a=0\Rightarrow x-x=-4ax\Rightarrow x=0$

218. $25x^2+9y^2-150x-90y-22=0\Rightarrow 25(x^2-6x)+9(y^2-10y)=-25$

$(h,k)=(3,5),e=4\Rightarrow\frac{x^2}{2}-\frac{y^2}{1}=0$

219. A point $y=x+c,bc(x,\alpha+c)$
Equation of chord of contact
$\frac{xkc}{2}-\frac{y(\alpha+c)}{1}=1\Rightarrow\frac{x}{2}\alpha-y\alpha-4c-1=0$

Equation is $\frac{x}{2}-y=0\ or\ cy+1=0$

Passing through ($x_1\Rightarrow\frac{x_1,y_1}{2}\alpha-y\alpha-4c-1=0$) $x_1-y_1\Rightarrow\frac{x_1}{y_1}=2$

220. distance of (0 , 0) to the line 2x+3y=6

$$OD = \frac{6}{\sqrt{13}} \quad \Delta OAD, \tan 45^0 = \frac{OD}{AD} \Rightarrow AD = OD = \frac{6}{\sqrt{13}}, AB = 2AD = \frac{12}{\sqrt{13}}$$

221. Section formula

222. H-orthocentre G centroid

S circumectre

223. $(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2) = (a_1a_2 + b_1b_2 + c_1c_2)$

$$\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} = \pm 1 \Rightarrow \cos 0^0 \ or \ \cos 180^0$$

224. Plane having (1, -1 ,0) and parallel to vector $i - 9j + k$

225. $L_1 = e^{\underset{x\to\infty}{Lt}\left[\frac{\sin x}{n} - 1\right]} \dfrac{\sin x}{x \sin x} = e^{-1}, L_2 = e^{\underset{x\to\infty}{Lt}(x-1)\left(\frac{1}{1-x}\right)} = e^{-1} \Rightarrow e(L_1 + L_2) = 2$

226. Put $\theta = \cos^{-1} x \Rightarrow \cos\theta = x, y = \text{Tan}^{-1}\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} \Rightarrow y = \dfrac{\pi}{2} - \dfrac{\theta}{2} \Rightarrow \dfrac{dy}{d\theta} = \dfrac{-1}{2}$

228. $\max\{x, x^3\} = \begin{cases} x & x < -1 \\ x^3 & -1 \le x < 0 \\ x & 0 < x \le 1 \\ x^3 & x \ge 1 \end{cases}$ not defferntiable at x=-1,0 , 1

229. $y = x^3 - 2x + 1, G = \dfrac{dy}{dx} = 3x^2 - 2, \quad gradient\ G = \dfrac{dy}{dx}$

$G = 3x^2 - 2, \dfrac{dG}{dt} = 6x.\dfrac{dx}{dt} - 2$

Put $x = 2 \Rightarrow \dfrac{dG}{dt} = 6 \times -dx/dt = 12\dfrac{dx}{dt}$

230. $y = 12x + b\ slope\ m = 12, \dfrac{dy}{dx} = 3x^2\ i.e 3x^2 = 12, x = \pm 2$

$y = 12x + b\ \ is\ \tan gent\ to\ \ y = x^3\ ,\quad 6 = \pm 16$

At (2 ,8) and (-2 , -8)

231. $f(x) = |p - q| + r|x|, where\ \ p, q\ and\ r > 0$

$$f(x) = (px - q) - rx \quad x < 0$$
$$= -(px - q) + rx \quad 0 \le x < p/q$$
$$= px + q + rx \qquad x \ge -p/q$$

232. $\dfrac{dx}{d\theta} = a(1 + \cos\theta), \dfrac{dy}{dx} = \dfrac{-\sin\theta}{1 + \cos\theta} = -\text{Tan}\,\theta/2$

$\dfrac{dy}{dx} = a(-\sin\theta)$

$\text{Tan}\,\theta = \dfrac{1}{\sqrt{3}} = \theta = 2\pi/3$

235. $\int f(x).\sin x\cos dx = \frac{1}{2(b^2-a^2)}\log f(xy)+c$ differentiable

$f(x).\sin x\cos x = \frac{1}{2(b^2-a^2)}\frac{1}{f(x)}f\left(\frac{1}{x}\right)$

237. $f\left(\frac{1}{x}\right)=8 \Rightarrow 2A+B=8, \int_0^1 f(x)dx = \frac{8}{3} \Rightarrow \left[\frac{Ax^3}{3}+\frac{Bx^2}{2}\right]_0^1 = \frac{8}{3}$

$\frac{A}{3}+\frac{B}{2}=\frac{8}{3}$ $A=2$ *and* $B=4$

238. $S_\alpha = \lim_{x\to\alpha}\frac{1}{r+\sqrt{m}} \Rightarrow \lim_{x\to\alpha^{\frac{-1}{n}}}\sum\frac{1}{\frac{r}{n}+\sqrt{\frac{r}{n}}}$

$\int_0^1 \frac{dx}{\sqrt{x}(\sqrt{x}+1)}dx$

Put $\sqrt{x}=t, 2\int_1^2 \frac{1}{t}dt = 2\log 2$

239. $x^2 = 9y.........(1)$

$y^2 = 2x.........(2)$

Area $\int_0^1(\sqrt{2x}-\frac{x^2}{2})dx+\int_1^{\sqrt{2}}(\sqrt{3-x^2}-\frac{x^2}{2})dx = \frac{\sqrt{2}}{3}+\frac{3}{2}\sin^{-1}(1/3)$

240. $xy^2dx+ydx+xdy+xydy+x^3y^2dy=0$

$(y\ dx+xdy)+xy(ydx+xdy)+x^3y^2dy=0$

$(1+xy)d(xy)+x^2y^2dy=0$

Divided by x^3y^3

$2t^2\log y-2t-1=c.2t^2 \Rightarrow 2x^2y^2\log y-2xy-1=2cx^2y^2$

241. Domain of $f(x)$ is $R-\{R\}$ $\because x-2\neq 0$

$x\neq 2$

Let $y=f(x)=\frac{x+3}{x-2}\Rightarrow xy-2y=x+3\Rightarrow x(y-1)=2y+3\Rightarrow x=\frac{2y+3}{y-1}$

Range of $f(x)$ is $R-\{1\}$ $\because y-1\neq 0\Rightarrow y\neq 0$

242. Given $f(30)=f(10\times 3)=\frac{f(10)}{3}=20$

$f(10)=3\times 20=60$

$f(40)=f(10\times 4)=\frac{f(10)}{4}=\frac{60}{4}=15$

243. $A=\begin{bmatrix}1 & -3 & 2\\ -2 & 1 & 3\\ 3 & 2 & -1\end{bmatrix}$ $|A|=1(-1-6)+3(2-9)+2(-4-3)=-7-21-4=-42$

$A^2(AdjA)=A.A(AdjA)=-42A$ $\qquad$ [$A(AdjA)=|A|A$

244. $\begin{vmatrix}1 & -3 & 1\\ 1 & 6 & 4\\ 1 & 3x & x^2\end{vmatrix}=0\Rightarrow 1(6x^2-12x)+3(x^2-4)+1(3x-6)=0$

$\Rightarrow x^2-x-2=0$

245. $A^3=\begin{bmatrix}x^3 & 0\\ x^2+x+1 & 1\end{bmatrix}=\begin{bmatrix}8 & 0\\ 7 & 1\end{bmatrix}\Rightarrow X^3=8=2^3\Rightarrow X=2$

246. if $a+a^{-1}=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}+\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}=I$

$\begin{bmatrix}2\sin\alpha & 0\\ 0 & 2\sin\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\Rightarrow 2\sin\alpha=1$

$\sin\alpha=\frac{1}{2}$

$\alpha=30^0=\frac{\pi}{6}$ g

247. $\cos\theta-\sin\theta=\sqrt{5}\sin\theta$

$\cos\theta=(\sqrt{5}+1)\sin\theta\frac{\sqrt{5}-1}{\sqrt{5}-1}\Rightarrow\sqrt{5}\cos\theta-\cos\theta=4\sin\theta$

$\cos\theta+4\sin\theta=\sqrt{5}\cos\theta$

248. $\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=90^0\Rightarrow\tan\frac{A}{2}+\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$

249. $\sin 22\frac{1}{2}^0=\sqrt{\frac{1-\cos 45^0}{2}}=\sqrt{\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)}=\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}\times\frac{\sqrt{2}}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}$

250. $(\sin\theta+\cos ec\theta)^2=4^2\Rightarrow\sin^2\theta+\cos ec^2\theta+2=16$

$\sin^2\theta+\cos ec^2\theta=16-2=14$

251. $\frac{1+\tanh x}{1-\tanh x}=\frac{1+\frac{e^x-e^{-x}}{e^x+e^{-x}}}{1-\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)}=\frac{e^x+e^{-x}+e^x-e^{-x}}{e^x+e^{-x}-e^x+e^{-x}}=\frac{2e^x}{2e^{-x}}=e^{2x}$

252. $s=\frac{a+b+c}{2}=\frac{9}{2}$ $\qquad$ $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\sqrt{\frac{\left(\frac{9}{2}-3\right)\left(\frac{9}{2}-4\right)}{\frac{9}{2}\left(\frac{9}{2}-2\right)}}=\frac{1}{\sqrt{15}}$

253. $r_1 \cot\frac{A}{2} + r_2 \cot\frac{B}{2} + r_3 \cot\frac{C}{2} = \sum r_1 \cot\frac{A}{2}$

$$\sum \frac{\Delta}{s-a} \times \frac{s(s-a)}{\Delta} = \sum s = s+s+s = 3s$$

254. $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\frac{2}{3}} = \frac{2}{\sin B} \Rightarrow \sin B = 1 \Rightarrow B = 90^0$

255. The equation of the plane. Perpendicular to the vector $3\bar{i} + \bar{j} + 5\bar{k}$ is $3x + y + 5z = d$

Given this is passing through $A(-2, 1, 3) \Rightarrow 3(-2) + 1 + 5(3) = d$

$$10 = d$$

Req. plane is $3x + y + 5z - 10 = 0$

$$ax + by + cz + d = 0$$

$a = 3,\ b = 1,\ c = 5,\ d = -10$

$\frac{a+b}{c+d} = \frac{3+1}{5-10} = \frac{4}{-5}$

256. $a = \lambda b$ $\quad \bar{a}.\bar{b} = 27$

$\lambda \bar{b}.\bar{b} = 27$

$\lambda |\bar{b}| = 27$

$\lambda(9+36+36) = 27$

$\lambda = \frac{27}{81} = \frac{1}{3}$

$\bar{a} = \lambda \bar{b} = \frac{1}{3}(3\hat{i} + 6j + 6k) = \hat{i} + 2j + 2k$

$|\bar{a}| = \sqrt{1+4+4} = \sqrt{9} = 3$

257. $\overline{BA} = \overline{OA} - \overline{OB} = 2\hat{i} + 4j + 8k$

Given $\overline{BA} \times \overline{F} = \begin{vmatrix} \hat{i} & j & k \\ 2 & 4 & 8 \\ 2 & 2 & 5 \end{vmatrix} = 4\hat{i} + 6j - 4k = 4\hat{i} + 6j + 2\lambda k$

$= -4 = 2\lambda \Rightarrow \lambda = -2$

258. $\bar{a}.\bar{b} = 60 \Rightarrow |\bar{a}||\bar{b}|\cos\theta = 60$

$13 \times 5 \cos\theta = 60 \Rightarrow \cos\theta = \frac{12}{13}$

$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}|\sin\theta = 13 \times 5 \times \frac{5}{13} = 25$

259. $13x + 43 = A(x+6) + B(2x+5)$

Put $x = -\frac{5}{2} \Rightarrow A = 3$

Put $x=-6 \Rightarrow B=5$

$A^2+B^2=9+25=34$

260. Put $x^{\frac{1}{3}}=t \Rightarrow t^2+t-2=0 \Rightarrow t^2+2t-t-2=0$

$t(t+2)-1(t+2)=0 \Rightarrow (t-1)(t+2)=0 \Rightarrow t=1,\ -2$

$x^{\frac{1}{3}}=1$ $\qquad$ $x^{\frac{1}{3}}=-2$

$x=1$ $\qquad$ $x=-8$

$\alpha^2+\beta^2=1^2+(-8)^2=65$

261. Let $y=\dfrac{x^2+x+1}{x^2-x+1} \Rightarrow yx^2-2y+y=x^2+x+1 \Rightarrow (y-1)x^2-(y+1)x+(y-1)=0$

For real $x,\ \Delta \geq 0$ $\quad (y+1)^2-4(y-1)^2 \geq 0 \Rightarrow 3y^2-10y+3 \leq 0$

$$y \in \left[\frac{1}{3},\ 3\right]$$

262. By synthetic division remainder $41x+3$

263. $\Delta=0 \Rightarrow (2m+1)^2-4m=0 \Rightarrow 4m^2+1=0$

$$m^2=-\frac{1}{4} \text{ (Which is not possible)}$$

$\therefore$ m has no real values

264. $(x-iy)^{\frac{1}{3}}=a-ib \Rightarrow x-iy=(a-ib)^3=\left(a^3-3ab^2\right)-i\left(3a^2b-b^3\right)$

$x=a^3-3ab^2,\quad y=3a^2b-b^3$

$\dfrac{x}{2a}+\dfrac{y}{2a}=\dfrac{a\left(a^2-3b^2\right)}{2a}+\dfrac{b\left(3a^2-b^2\right)}{2b}=2\left(a^2-b^2\right)$

265. $\left(-1+i\sqrt{3}\right)^{60}=2^{60}\left[-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}\right]^{60}=2^{60}\times\omega^{60}=2^{60}\left(\omega^3\right)^{20}=2^{60}(1)=2^{60}$

266. $|Z|+|Z-1| \geq |Z-(Z-1)|=1$

$|Z|+|Z-1| \geq 1$

Minimum value is 1

267. Arg $Z_2=-Arg\overline{Z_2}=-\dfrac{\pi}{5}$

$\therefore$ Arg $Z_1+ArgZ_2=\dfrac{\pi}{3}-\dfrac{\pi}{5}=\dfrac{2\pi}{15}$

268. Number of ways of selecting 4 oranges is 4+1=5

Number of ways of selecting 5 apples is 5+1= 6

Number of ways of selecting 7 mangoes is 7+1= 8

$\therefore$ The required number of ways $=5\times6\times8-1=239$

269. ${}^{(n-1)}C_3+{}^{(n-1)}C_4>{}^{n}C_3$

${}^{n}C_4>{}^{n}C_3 \Rightarrow n>3+4 \Rightarrow n>7$

$\therefore$ The least value of n is 8

270. Required number of triangles $= {}^{10}C_3 - {}^{6}C_3 = 120 - 20 = 100$

271. MULTIPLE has 8 letters, with 5 consonants and 3 vowels (L repeated twice)
The vowels in 2nd, 5th and 8th place can be arranged in 3! Ways = 6 ways
The remaining 5 consonants can be arranged in $\frac{5!}{2!} = \frac{120}{2}$
= 60 ways
Total number of wats $= 6 \times 60 = 360$

272. $\left[\frac{n}{l.c.m(4,5)}\right] + 1 = \left[\frac{100}{20}\right] + 1 = 5 + 1 = 6$

273. $1 + x = 1 + \frac{1}{5} + \frac{1.3}{1.2}\left(\frac{1}{5}\right)^2 + ...$

$(1 - X)^{\frac{-p}{q}} = 1 + \frac{p}{1}\left(\frac{x}{q}\right) + \frac{p(p+q)}{1-2}\left(\frac{X}{q}\right)^2 + ...$

Comparing $P = 1$ $\qquad p + q = 3 \Rightarrow 1 + q = 3 \Rightarrow q = 3 - 1 = 2$

$\frac{X}{q} = \frac{1}{5} \Rightarrow \frac{x}{2} = \frac{1}{5} \Rightarrow x = \frac{2}{5}$

$1 + x = \left(1 - \frac{2}{5}\right)^{\frac{-1}{2}} = \left(\frac{3}{5}\right)^{\frac{-1}{2}} = \left(\frac{5}{3}\right)^{\frac{1}{2}}$

$(1+x)^2 = \frac{5}{3} \Rightarrow 1 + x^2 + 2x = \frac{5}{3}$

$3 + 3x^2 + 6x = 5$

$3x^2 + 6x = 5 - 3 = 2$

$3x^2 + 6x = 2$

274. $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{84000}{20} - \left(\frac{1000}{20}\right)^2 = 4200 - 2500 = 1700$

275. Range = Highest value – lowest value = 35-7 = 28

276. $P(A \cup B) = P(A) + P(B) - P(A).P(B)$

$0.44 = 0.3 + x - (0.3)x$

$0.44 - 0.3 = (0.7)x$

$0.14 = (0.7)x$

$x = \frac{0.17}{0.7} = 0.2$

277. $P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$

278. Req. probability $= P(P \cap R^c) + P(P^c \cap R)$

$$=\frac{3}{4}\times\frac{1}{5}+\frac{1}{4}\times\frac{4}{5}=\frac{7}{20}$$

279. Mean $= np = 15$ variance $= npq = 10$

$q=\frac{npq}{np}=\frac{10}{15}=\frac{2}{3}$ $\quad\quad$ $p=1-q=1-\frac{2}{3}=\frac{1}{3}$

$$np=15\Rightarrow n\left(\frac{1}{3}\right)=15\Rightarrow n=15\times3=45$$

280. $P(x=2)+P(x=3)$

$$=\frac{e^{-3}\times3^2}{2!}+\frac{e^{-3}3^3}{3!}=e^{-3}\left[\frac{9}{2}+\frac{27}{6}\right]=9e^{-3}$$

281. $r=\frac{\sqrt{5}}{\sqrt{2}}$

Eq. of circle is $(x-2)^2+(y+1)^2=\frac{5}{2}$

Let $y=mx$ be other tangent

282. $c_1c_2=r_1-r_2$

283. $r=d$

284. $c=(1,3),\ r=2$

$c_1=(2,1)$

$r=\sqrt{(2-1)^2+(1-3)^2+2^2}=4$

285. The point $\left(\sqrt{3}+\frac{r}{2},\frac{r\sqrt{3}}{2}\right)$ lies on $y^2=x+2$

286. $a=3,\ b=2,\ sp=4$

$\frac{1}{sp}+\frac{1}{s'p}=\frac{2}{l}\Rightarrow s'p=\frac{4}{5}$

$s'p+s'p'=6$

288. $\frac{\left(\frac{3x+y-1}{\sqrt{9}+1}\right)^2}{9}+\frac{\left(\frac{x-3y+3}{\sqrt{1+9}}\right)^2}{6}=1$

289. 2A$=H+C$ $\quad\quad$ $H: xy+3x-4y+13=0$

$A: xy+3x-4y+k=0$

290. Eq. of tangent is $3x\sec\theta-4y\tan\theta-12=0$

$$\frac{h-5}{3\sec\theta}=\frac{k-0}{-4\tan\theta}=\frac{-(15\sec\theta-12)}{9\sec^2\theta+16\tan^2\theta}$$

$$\frac{h-5}{3\sec\theta}=\frac{k}{-4\tan\theta}=\frac{-3}{5\sec\theta+4}$$

291. $a=2,\ b=\sqrt{3},\ e=\dfrac{1}{2}$

Focus $s'=(\pm ae,0)=(\pm 1,0)$

$2a'=2\sin\theta \quad a'e'=\sin\theta e'=1\Rightarrow e'=\dfrac{1}{\sin\theta}$

292. $I=\int(1+\tan^2 x)(1+\tan^4 x)dx$

$=I=\int(1+\tan^2 x)\sec^2 x dx$

293. put $3^{3^x}=t$

294. $\int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+c$

295. standard result

296. put $\sin x-\cos x=t$

297. $I=\int_0^{\frac{\pi}{6}}\cos^6 3\theta\sin^2 3\theta d\theta$

Put $3\theta=t$

298. $I=\int_0^{\pi}\left|2\sin\dfrac{x}{2}-1\right|dx$

$I=\int_0^{\frac{\pi}{3}}-\left(2\sin\dfrac{x}{2}-1\right)dx+\int_{\frac{\pi}{3}}^{\pi}\left(2\sin\dfrac{x}{2}-1\right)dx$

299. $\left(\dfrac{d^4y}{dx^4}+\dfrac{d^2y}{dx^2}\right)^3=\left(a\dfrac{d^3y}{dx^3}\right)^2$

300. $\sin(A+B)+\sin(A-B)=2\sin A\cos B$

301. put $y=vx$

302. $\dfrac{dx}{dy}-\dfrac{x}{y}=2y^2$

303. If $x=1,y=1$ then $f(2)-k=f(1)+2\Rightarrow k=4$

If $x=0,y=1$ then $f(1)-0=f(0)+2\Rightarrow f(0)=0$

If $y=x$ then $f(2x)-4x^2=f(x)+2x^2\Rightarrow f(2x)-f(x)=6x^2$

$\therefore f(20)-f(10)=600$

304. By using L hospatal

305. Since $x\to 2+\therefore[x]=2$

Also $\left[\dfrac{x}{3}\right]=\left[\dfrac{2}{3}\right]=0$ $\quad\therefore$ Given limit $\dfrac{8}{3}-0=\dfrac{8}{3}$

306. $f(x)$ is discontinuous at all integers except ‘1’.

307. $x+y+xy=0$

308. The slope of horizontal line is 0

309. $f'(x) < 0$

310. $f'(x) = 0$

311. Centroid of ΔABC = Centroid of ΔPQR

312. $\alpha - p = \frac{a(x-1)}{2}$

313. Externally divide ratio $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

314. $(2a+3b)^2 = c^2$

315. Given line is parallel to x-axis $\Rightarrow x$ coefficient = 0

316. Equation of the directrices are $x = h \pm a/e$

317. Mid point of $\overleftrightarrow{AC}$ = Mid point of $\overleftrightarrow{BD}$

318. $D = \left(\frac{2k}{k+1}, \frac{-3k-11}{k+1}, \frac{k+4}{k+1}\right)$

319. Dr's of $\overleftrightarrow{AB}(4, -3, 4)$

PHYSICS

1. $\frac{\Delta l}{l} \times 100 = \frac{1}{3}\left(\frac{\Delta m}{m} \times 100 + \frac{\Delta \rho}{\rho} \times 100\right)$

2. $h = \frac{1}{2}gt^2 - ut; time\ of\ ascent = \frac{u}{g}$

3. $V_{\text{arg}} = \frac{\int_0^2 v dt}{\int_0^2 dt}$

4. $R = \frac{u^2 \sin 2\theta}{g}$

5. $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g; h_{\max} = \frac{u^2}{2g}$

6. $W_{all\ force} = \Delta KE$

7. $e = \frac{v_2 - v_1}{u_1 - u_2}$

8. $a = \frac{g \sin \theta}{\left(1 + \frac{k^2}{R^2}\right)}$

9. The required energy for this work is given by

$$\frac{GMm}{R} = mgR \Rightarrow 1000\times10\times6400\times10^{3} = 6.4\times10^{10}\,J$$

10. $TE = \frac{1}{2}m\omega^2 A^2$

11. $F = \frac{YAe}{L}$

12. work done all forces= change in KE

13. $PV = \mu RT \Rightarrow PV = \frac{m}{m}RT \Rightarrow PM = \rho RT \Rightarrow P\alpha\rho$

14. Conceptual

15. $V_{rms}\alpha\sqrt{T}$

16. $\lambda = \frac{1}{\sqrt{2}\pi nd^2}$

17. Conceptual

18. $TV^{r-1} = cons\tan t; n = 1-\frac{T_2}{T_1}$

19. $\left(\frac{Q}{t}\right)_I = \left(\frac{Q}{t}\right)_{II} + \left(\frac{Q}{t}\right)_{III}$

20. $shift = \frac{md}{M+m}$

21. 9 nodes means string vibrates in 10th harmonic $l = \frac{10\lambda}{2} \Rightarrow \lambda = \frac{2l}{10} \Rightarrow f = \frac{v}{\lambda}$

22. $\mu = \frac{\sin i}{\sin r} \Rightarrow \frac{1}{\sin e}$

23. $\frac{f^1}{f} = \frac{(\mu_{lens}-1)}{\left(\frac{\mu_{lens}}{\mu_{liq}}-1\right)}$

24. Conceptual

25. Conceptual

26. $U = \frac{q^2}{2C}; C_{eq} = \frac{C}{1+C}$

$$= \frac{(80\times10^{-6})^2}{2\times2\times10^{-6}}; q = C_{eq}V \Rightarrow U = 1600\mu J$$

27. $I = \frac{V}{R_{eff}}$

28. $\tan\delta = \frac{B_v}{B_H} = \sqrt{3}$

29. $B = \mu_0 ni \Rightarrow \frac{B_2}{B_1} = \frac{n_2}{n_1}\times\frac{i_2}{i_1} \Rightarrow B_2 = \frac{B_1\,n_2\,i_2}{n_1\,i_1}$

30. $B = B_1 + B_2$

31. $e = \frac{Bwl^2}{2} \Rightarrow \frac{\pi}{3000} = \frac{5\times10^{-5}\times2\pi f}{2} \Rightarrow f = 400\, cycles\ per\ \min ute$

32. $P = I_{rms}\, V_{rms}\, \cos\phi \Rightarrow \frac{V_{rms^2}}{Z}\times\frac{R}{Z} \Rightarrow P = \frac{V_0^{\ 2}}{2}\times\frac{R}{z^2}$

33. $h\nu = \phi + K_{\max}$

34. conceptual

35. $E_2 = \frac{13.6}{(2)^2} eV = 3.4eV$

36. Conceptual

37. $235_{U_{92}} \rightarrow {}^{206}_{82}U$

38. Conceptual

39. $\mu = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$

40. using Kirchoff's law

41. $n_1 u_1 = n_2 u_2$

$1m^2 = n_2 \times xm \times xm$

$1m^2 = n_2\, x^2 m^2$

$\Rightarrow n_{02} = \frac{1}{x^2}$

42. time of fall of ball $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times45}{10}} = 3\sec$

In 3s boat covers 12m

Speed $= \frac{dis\tan ce}{time} = \frac{12}{3} = 4m/s$

43. $H = ut - \frac{1}{2}gt^2$

$gt^2 - 2ut + 2H = 0$

Sum of roots $t_1 + t_2 = \frac{2u}{g}$

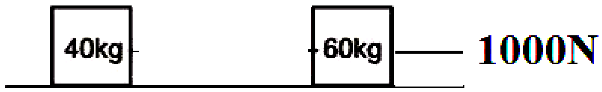
$\Rightarrow u = g\frac{\left(t_1 + t_2\right)}{2}$

$H_{\max} = \frac{u^2}{2g} = g\frac{\left(t_1 + t_2\right)^2}{8}$

44. w=constant

` v and a change in direction displacement changes both in magnitude and direction

45. Taking both mass of single system we get



1000=(40+60) a

$\rightarrow a = 10 m/s^2$

For 60 kg block

1000-T=60a

1000-T=600 $\Rightarrow T = 400N$

46. 25% of energy -2cm

75% of energy -6 cm

47. P= $\dfrac{(M+m)gh}{t}$

$$\Rightarrow 1568 = \frac{(M+50)\times 9.8\times 40}{25}$$

$M = 50kg$

48. $\omega_1 = 60\times\dfrac{2\pi}{60} = 2\pi rad/s$

$$\omega_2 = 360\times\frac{2\pi}{60} = 12\pi\, rad/s$$

K.E= $= \dfrac{1}{2} I\left(\omega_2^2 - \omega_1^2\right)$

$$484 = \frac{1}{2} I\left[(12\pi)^2 - (2\pi)^2\right]$$

$$484 = \frac{1}{2} I(1440 - 40)$$

$I = 0.7 kgm^2$

49. K.E = $\dfrac{1}{2}mv^2\left(1+\dfrac{K^2}{R^2}\right)$

$$\frac{KE_{ring}}{KE_{disc}} = \frac{\frac{1}{2}mv^2(1+1)}{\frac{1}{2}mv^2\left(1+\frac{1}{2}\right)}$$

$$\frac{8}{K.E_{disc}} = \frac{2}{\frac{3}{2}}$$

$KE_{disc} = 6J$

50. Maximum velocity $v_{\max} = A\omega = 100\times 6\times 10^{-2} = 6m/s$

Max K.E $= \dfrac{1}{2}mv_{\max}^2$

$$= \frac{1}{2}\times 1\times 6^2 = 18J$$

51. $kx =$ constant

$$kx = k'\left(\frac{x}{3}\right)$$

$$\Rightarrow k' = 3k$$

52. $T_A = 8T_B$

$$\Rightarrow T_A^2 = 64T_B^2$$

$$\Rightarrow RA^3 = 64R_B^3$$

$$\Rightarrow RA = 4R_B$$

Now $V = \dfrac{2\pi R}{T}$

$$\frac{V_A}{V_B} = \frac{R_A}{T_A} \times \frac{T_B}{R_B}$$

$$= \frac{R_A}{R_B} \times \frac{T_B}{T_A} = 4 \times \frac{1}{8} = \frac{1}{2}$$

53. $m_1 = m$ $\quad\quad m_2 = 3m$

Distance of the Neutral point from $m = x$

$$x = \frac{l}{1\sqrt{\frac{m_2}{m_1}}} = \frac{d}{1\sqrt{\frac{3m}{m}}} = \frac{d}{1+\sqrt{3}}$$

$$x = \frac{d}{1+\sqrt{3}}$$

54. Let l =natural length

$e_1 = l_1 - l \quad e_2 = l_2 - l$

$e_1 = \dfrac{T_1 l}{y_A} \quad e_2 = \dfrac{T_2 l}{y_A}$

$$\frac{e_1}{e_2} = \frac{T_1}{T_2} \Rightarrow \frac{l_1 - l}{l_2 - l} = \frac{T_1}{T_2}$$

$$l = \frac{l_1 T_2 - l_2 T_2}{T_2 - T_1}$$

55. Pascal’s law is used in hydraulic lift

56. $\Delta u = uf - ui$

$$= \left[T\left(4\pi(2T)^2\right)\right] - T\left[4\pi R^2\right] \times 2$$

$$= 32\pi R^2 T - 8\pi R^2 T$$

$$= 24\pi R^2 T$$

57. Absorption efficient = amount of absorb radiation (Q_a)/amount of incident radiation (Q_i)

And for a perfect black body $Q_a = Q_i$

$\therefore a = 1$

58. $m_sL_s + m_ss(t_1 - t) = m_cL_c + m_cs(t - 0)$

$10(540) + 10(1)(100 - t) = 50(80) + 50(1)(t - 0)$

$\Rightarrow t = 40^0 C$

59. $\eta = 1 - \frac{300}{600} = 1 - \frac{1}{2} = \frac{1}{2}$

Now , $w = \eta Q = \frac{1}{2} \times 800 = 400J$

61. average kinetic energy $E = \frac{3}{2}K_{BT}$

$K \alpha T$

At same temperature average kinetic energy of O_2 molecules will be same as E

62. $w =$ are of ABC

$= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times (3v - v)(3p - p)$

$= 2pv$

63. The standard traverse wave is

$y = A\sin(\omega t - kx)$

$\omega = 30$ and $k = 40$

On comparing we get

$v = \frac{w}{k} = \frac{30}{40} = 0.75m/\sec$

64. $\sin C = \frac{\mu_2}{\mu_1}$

$\sin C = \frac{1}{2}$

$C = 30^0$

65. $y = \frac{m_1\lambda_1 D}{d} = \frac{780mD}{d}$

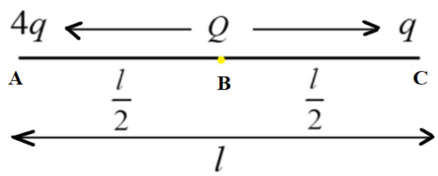
$y = \frac{(m+1)\lambda_2 D}{d} = \frac{520(m+1)D}{d}$

$780m = 520(m+1)$

$260m = 520$

$m = 2$

66. The force exerted on the charge q, Q is given as follows.



Resultant force on charge ‘q’ is zero

$$k = \frac{4q.q}{l^2} + \frac{kQq}{\left(\frac{l}{2}\right)^2} = 0$$

$$\frac{4q}{l^2} + \frac{4Q}{l^2} = 0$$

$$\Rightarrow q + Q = 0$$

$$Q = -q$$

Hence, Q should be -q in order to make the net force on q to be zero.

67. Three capacity are in parallel

$$C_{cq} = c_1 + c_2 c_3 = 2 + 4 + 6 = 12\mu F$$

68. $$v = \frac{k_q}{r}$$

$$q = \frac{v \times r}{k} = \frac{25 \times 10^3 \times 9 \times 10^{-2}}{9 \times 10^9}$$

$$= 25 \times 10^{-9} c = 0.25 \mu c$$

69. $$1.8 = 17(0.06 + r)$$

$$\Rightarrow \frac{1.8}{1.7} = 0.06 + r$$

$$\Rightarrow r = \frac{1.8}{17} - 0.06 = 0.046\Omega$$

71. $$B = \frac{\mu_0 i}{2\pi r}$$

$$B \alpha \frac{1}{r}$$

$$B_{r/2} = 2T$$

$$B_{2r} = \frac{1}{2} T$$

$$B_{3r} = \frac{1}{3} T$$

72. According to modified ampere’s (or) Maxwell’s ampere law

$$\overline{V} \times \overline{B} = \mu_0 \left(\overline{J} + \overline{J_d}\right)$$

$$\overline{J_d} = \varepsilon_0 \frac{d\overline{E}}{dt}$$

73. $E = \frac{2BAn}{t}$

$= \frac{2 \times 500 \times 10^{-6} \times 200}{20 \times 10^{-3}}$

$E = 10 volt$

75. $i_{rms} = \frac{v_{rms}}{R} = \frac{200}{10 \times 10^3} = 2.2 mA$

76. $\frac{1}{\lambda_{BL}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36} R \Rightarrow \lambda_B = \frac{36}{5R} - (1)$

$\frac{1}{\lambda_{PL}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{7}{144} R \Rightarrow \lambda_P = \frac{144}{7R} - (2)$

$\frac{\lambda_{Bl}}{\lambda_{Pl}} = \frac{7}{20}$

77. $KE_{max} = E - W_0 = \frac{hc}{\lambda} - W_0$

$= \frac{12400 evA^0}{5000 A^0} - 1.9 ev$

$= 0.58 ev$

78. Given Half life of sample = 103 years

Initial amount NO=100g

Final amount N=3.125g

$\frac{N_o}{N} = 2^n = \frac{100}{3.125}$

$\Rightarrow 2^n = 32 \Rightarrow n = 5$

Time taken $= 103 \times 5 = 515 years$

79. $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{\Delta I_C}{\Delta I_E - \Delta I_C}$

$\beta = \frac{8.9}{9 - 8.9} = \frac{8.9}{0.1} = 89$

80. coverage range of TV tower is given by

$d = \sqrt{2Rh} = \sqrt{2 \times 64 \times 10^6 \times 1160} = 452548 = 45255 m$

81. $\frac{\Delta P}{P} \times 100 = \frac{1}{2}\left[\frac{\Delta a}{a} \times 100\right] + \frac{1}{2}\left[\frac{\Delta b}{b} \times 100\right] + \alpha\left[\frac{\Delta d}{d} \times 100\right] + \frac{1}{2}\left[\frac{\Delta c}{c} \times 100\right]$

$2 = \frac{1}{2}\left[\frac{1}{2}\right] + \frac{1}{2}\left[\frac{1}{2}\right] + \alpha\left[\frac{1}{2}\right] + \frac{1}{2}\left[\frac{1}{2}\right] \Rightarrow \alpha = \frac{5}{2}$

82. $\frac{dv}{dt} = 4t - 8 \Rightarrow a = 4t - 8; at\ t = 5 \sec, a = 12 ms^{-2}$

83. Distance covered by the stone in5 s is $4t + \frac{1}{2} gt^2 = 0 \times 5 + \frac{1}{2} g(5)^3 = \frac{25g}{2} ------(1)$

Distance travelled in n^{th} second $\frac{g}{2}(2n-1)$--------- (2) $\frac{25g}{2}=\frac{g}{2}(2n-1)\Rightarrow n=13$

84. $\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=30^0\Rightarrow\frac{q}{2u^2\cos^2\theta}=\frac{1}{60}=20m/s$

85. $F=(m_1+m_2)a\Rightarrow a=\frac{F}{m_1+m_2};T=m_1a\Rightarrow\left(\frac{m_1}{m_1+m_2}\right)F$

86. $W=\Delta U=\int_u^{l/6}\frac{mg}{l}\times dx=\frac{mgl}{72}$

87. Conceptual

88. By law of conservation of mechanical energy $V_0=\sqrt{15}\,V$

89. $B=\frac{-P}{\left(\frac{\Delta V}{V}\right)}=\frac{8\times10^8}{\left(\frac{20}{100}\right)}=4\times10^9\,N\;m^{-2}$

90. $V_T=\frac{2gr^2(\rho-\sigma)}{9\eta};W=\frac{1}{2}mV^2{}_T\Rightarrow W\alpha r^7$

91. Heat gain=Heat loss
50 s (T-10)= 50 s (100-T)

92. $\Delta W=$Area enclosed with in $P-v$ diagram

93. Conceptual

94. $\sqrt{\frac{3R(20)}{2}}=\sqrt{\frac{3R(T)}{32}}\Rightarrow T=320K$

95. $T=2\pi\sqrt{\frac{l}{g+a}}\Rightarrow\frac{\pi}{\sqrt{3}}s$

96. $N=6,AN=5$

97. Conceptual

98. As $w=q[v_1-v_2]=0$

99. Conceptual

100. $r=\left(\frac{l_1-l_2}{l_2}\right)R\Rightarrow\left(\frac{560-500}{500}\right)10\Rightarrow1.2\Omega$

101. Conceptual

102. Concptual

103. In a cyclotron,frequency of rotation of a charged particle
$f=\frac{Bq}{2\pi m}\Rightarrow f\alpha\frac{Bq}{m}$

104. Magnetic field at C=magnetic field at C
Due to straight wire + magnetic field due to semi circular ring
$\frac{\mu_0 i}{4\pi r}+\frac{\mu_0 i}{4r}\Rightarrow\frac{\mu_0 i}{4\pi r}[1+\pi]$

105. $M^1 = \frac{2M\sin(\theta/2)}{4\pi} \Rightarrow M^1 = \frac{2M}{\pi}$

106. $\mu_r = 1 + \varphi_B \Rightarrow 5499$

107. As per Faraday's law $e = \frac{-d\phi}{dt}$

108. $Z = \sqrt{R^2 + \omega^2 L^2} \Rightarrow \sqrt{(8)^2 + (2\pi \times 50 \times \frac{60}{\pi} \times 10^{-3})^2} = 10\Omega$

109. $B_0 = \sqrt{\frac{2\mu_0 I}{c}} \Rightarrow \frac{\sqrt{2 \times 4\pi \times 10^{-7} \times 9240}}{3 \times 10^8} = 8.8\mu T$

110. $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \Rightarrow \frac{3}{20} = (\mu - 1)(\frac{1}{5} - \frac{1}{(-10)}) \Rightarrow \mu = 1.5$

111. $\sin\theta_c = \frac{n_2}{n_1} = \frac{\sqrt{3}}{2} = 60^0$

112. As point source emits light in every direction. So if we draw a secondary wave front and join along its
tangent we will get sphere.

113. $n_1\lambda_1 = n_2\lambda_2 \Rightarrow n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{60 \times 6600}{4400} = 90 \; fringes$

114. $\lambda = \frac{h}{\sqrt{2mKE}} = \frac{h}{\sqrt{2mE}}; \lambda \propto \frac{1}{\sqrt{mE}}$

$\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{(2m)(2E)}}{\sqrt{mE}} \Rightarrow \lambda_2 = \frac{\lambda_1}{2} = \frac{\lambda}{2}$

115. Conceptual

116. Conceptual

117. Conceptual

118. $\beta = \frac{\Delta I_c}{\Delta I_B} = \frac{6.8}{0.2} = 34$

119. $y = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}}.\overline{\overline{B}} = A.B$

Gate is AND gate

120. $m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} = \frac{16-4}{16+4} = 0.6$

121. Use principle of homogeneity

122. $\sqrt{2gh}$

123. $b\cos\theta = \frac{\vec{a}.\vec{b}}{a}$

124. $\overrightarrow{V_{rm}} = \vec{V}_r - \vec{V}_m \; and \; \left|\overrightarrow{V_{rm}}\right| = \sqrt{V_r^2 + V_m^2}$

125. $m_2 g - T = m_2 a; T - m_1 g\sin\theta = m_1 a$

126. $F_1 = mg(\sin\theta - \mu_k\cos\theta); F_2 = mg(\sin\theta + \mu_k\cos\theta)$

127. $W=\frac{1}{2}K\left(x_2^2-x_1^2\right)$

128. $\tan\theta=\frac{a_c}{a_t}=\frac{v^2/r}{g\sin\theta}$

129. $\alpha=\frac{d\omega}{dt}$

130. $I_1\omega_1=I_2\omega_2; I_1=I; I_2=I+mR^2$

131. $v_{\max}=\omega A \,....(i)$

$a_{\max}=\omega^2 A \,....(ii)$

$\frac{a_{\max}}{v_{\max}}=\frac{\omega^2 A}{\omega A}=\omega \quad \therefore \frac{a_{\max}}{v_{\max}}=\frac{2\pi}{T}$

$T=2\pi\left(\frac{v_{\max}}{a_{\max}}\right); \; \therefore T=2\pi\left(\frac{30cms^{-1}}{60cms^{-2}}\right)=\pi s$

132. $V_e=\sqrt{\frac{2GM}{R}} \Rightarrow \frac{v_1}{v_2}=\sqrt{\frac{M_1}{M_2}\frac{R_2}{R_1}}$

133. $Y=\frac{Fl}{Ae}; \frac{e_1}{e_2}=\frac{F_1 l_1 r_2^2 Y_2}{F_2 l_2 r_1^2 Y_1}$

134. $\Delta P=\frac{2T}{r}$

135. $\gamma_R=\gamma_A+\gamma_g$

136. $dU=dQ+dW$

137. $dU=mc_v dR$

138. $\frac{W}{Q_1}=1-\frac{T_2}{T_1}$

139. $V_{rms}=\sqrt{\frac{v_1^2+v_2^2+v_3^2+v_4^2}{4}}$

140. $T=2\pi\sqrt{\frac{m}{k}}$

141. $V_p=A\omega\cos Kx\cos\omega t$

142. ${}_w\mu_g=\frac{\mu_g}{\mu_w};$

143. $y_2=5\left(\frac{\lambda_2 D}{d}\right); y_1=10\left(\frac{\lambda_1 D}{d}\right)$

144. $E=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+----\right]$

145. $Q=nCV$

146. $i = \frac{q}{t}$

147. $v = iR,\ i = \frac{E}{R+r}$

148. $\mu_0 \sum i$

149. $R = \frac{mv}{Bq} \Rightarrow v\alpha \frac{q}{m}$

150. For semicircle, $M^1 = \frac{2M}{\pi}$

151. $e = -M\left(\frac{di}{dt}\right)$

152. $V = 120\sin 100\pi t \cos 100\pi t$
$= 60 \times 2\sin 100\pi t.\cos 100\pi t$
$= 60\sin 200\pi t$ and $\therefore \omega = 200\pi$
$\therefore n = 100cps$

153. $\lambda = \frac{h}{\sqrt{2mE}}$

154. Energy required $E_\alpha - E_{10} = \frac{13.6}{10^2} = 0.136eV$

155. $2nHz$

156. $Y = \overline{A+B}$ is the output of NOR gate

157. $P = \frac{xN_A}{A} \times \frac{E}{t}$

158. $n = \frac{1}{2[\sqrt{LC}}, \lambda = \frac{C}{n}$

159. By using $f_c \approx 9\left(N_{\max}\right)^{\frac{1}{2}}$
; $f_c \approx 9\left(10^{12}\right)^{\frac{1}{2}}$

160. $n_1 = 250\,Hz,\ n_2 = ?,\ \Delta n = 5beats/s$ on loading wax n_1 decreases $\left(n_1 < n_1\right)$ and $\Delta n^{'} = beats/s \Rightarrow \Delta n^{'} < \Delta n$ this is possible when $n_1 - n_2 = \Delta n$

CHEMISTRY

1. $r = \frac{n^2}{2} \times 0.529A^0$

2. $ionic\ size\ \alpha \frac{1}{nuclear\ charge}$ *for iso electrone species*

3. E_A *order* $S > Se > O$

4. Conceptual

5. $SF_4(\mu \# 0), XeF_4(\mu = 0), SiF_4(\mu = 0), BF_3(\mu = 0)$
6. Conceptual
7. Conceptual
8. Conceptual
9. It results in lowering of the level of oceans
10. b measures volume of molecules
11. Conceptual
12. No change in colour
13. $P^H = P^{ka} + \log \frac{[s]}{[a]}$
14. $11.2V - 2N$
$?-1.5N$
15. No does not react with N_2 directly
19. Conceptual
20. Conceptual
21. Conceptual
22. Conceptual
23. $p = K_H.X$
24. Conceptual
25. Conceptual
26. Conceptual
27. Conceptual
28. $excess\ of\ KI \Rightarrow AgI / I^-$
29. Conceptual
30. Conceptual
31. $I^{1-} \xrightarrow{H^+} I_2, I^- \xrightarrow{neutral} IO_3^-$
32. Conceptual
33. $ZnO_2 + H_2O \rightarrow HNO_2 + HNO_3$
34. Conceptual
35. All are correct
36. $C^{(+)}$ ion sterilizing will react fasters
37. CCl_3CHO
38. Nucleophile addition reaction reactivity decrease with donor group
39. Conceptual
40. Benzoic acids
41. Energy of electron in nth orbitals is given by

$$E_n = -2.18 \times 10^{-18} \frac{Z^2}{n^2} J$$

For helium ion $\left(He^+\right)$

$Z = 2,\ n = 1$

$E_n = -2.18\times10^{-18}\times\frac{2^2}{1^2}$

For lithium ion (Li^{+2})

$Z = 3,\ n = 3$

$E_n = -2.18\times10^{-18}\times\frac{3^2}{3^2} = -2.18\times10^{-18}J$

42. $\frac{hv_1 - hv_0}{hv_2 - hv_0} = \frac{K.E_1}{K.E_2}$

$\frac{h(1.6\times10^{16} - v_0)}{h(1.0\times10^{16} - v_0)} = \frac{2K.E}{K.E}$

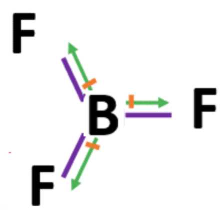
$v_0 = 1.6\times10^{16} - 2(1.0\times10^{16})$

$v_0 = 4\times10^{15}Hz$

43. Mn_2, O_7, CrO_3, V_2O_5 are acidic

Due to higher oxidation state of central metal oxides

44. I) In BF_3 electronegativity of F-atom is greater than B



II) Covalent bond length increases as

$C-O > N-O > O-H$

Due to increase in size of atom

III) bond order $= \frac{N_b - N_a}{2}$

45. $C_2 = \sigma1s^2 *1s^2\sigma2s^2 *2s^2\pi2p_x^2\pi2p_y^2$

$B.O = \frac{8-4}{2} = 2$

$B_2 = \sigma1s^2 *1s^2\sigma2s^2 *2s^2\pi2p_x^1\pi2p_y^1$

$B.O = \frac{6-4}{2} = 1$

$He_2 = \sigma1s^2\sigma*1s^2$

$B.O = \frac{2-2}{2} = 0$

Hence, bond order

$C_3 > B_2 > He_2$

46. K.E $\frac{3}{2}RT$ (for mole)

$u_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{2M}} \times 2$

$= \sqrt{\frac{2K.E}{M}}$

$= \frac{1}{2} \times 10^3 = 5 \times 10^2 m/\sec$

$= 5 \times 10^4 cm/\sec$

47. $K_2Cr_2O_7 + 3Na_2SO_3 + 4H_2SO_4 \rightarrow K_2SO_4 + CT_2\left(SO_4\right)_3 + 3Na_2SO_4 + 4H_2O$

$K_2Cr_2O_7$ Na_2SO_3

No of moles = 0.1 mole no. of mole = 0.15

3 moles of $Na_2SO_3 = 0.15$

1 mole of $Na_2SO_3 = \frac{0.15}{3} = 0.05 moles$

No. of mole of $K_2Cr_2O_7$ remained $= 0.1 - 0.05 = 0.05 moles$

48. $n = \frac{wt}{GAw} = \frac{54}{27} = 2 moles$

Heart required $(Q) = nC\Delta T = 2 \times 24 \times 20$

$= 960 J$

49. $\underset{(g)}{N_2} + \underset{(g)}{O_2} \rightleftharpoons 2NO_{(g)}$

$K_c = \frac{[NO]^2}{[N_2][O_2]}$

Given $[N_2] = [O_2]$

$0.5625 = \frac{\left[3 \times 10^{-3}\right]^2}{[N_2]^2}$

$[N_2]^2 = \frac{\left[3 \times 10^{-3}\right]^2}{0.5625} = 16 \times 16^{-6}$

$[N_2] = 4 \times 10^{-3}$

50. $A_2B \rightleftharpoons \underset{2B}{2A^+} + \underset{S}{B^{-2}}$

$K_{sp} = \left[A^+\right]^2 \left[B^{-2}\right]^1$

$= (2s)^2 (s)$

$= 4S^3$

$K_{sp} = 4s^3$

$3.2 \times 10^{-11} = 4s^3$

$8 \times 10^{-12} = s^3$

$s = 2 \times 10^{-4}$

51. Volume strength $N \times 5.6$

$N = \frac{v.s}{5.6} = \frac{20}{5.6} = 3.57N$

$KMnO_4$ in acidic medium

$MNO_4^- + 5e^- \rightarrow Mn^{+2}$

$N_1 v_1$ of $H_2O_2 = N_2 v_2$ of $KMnO_4$

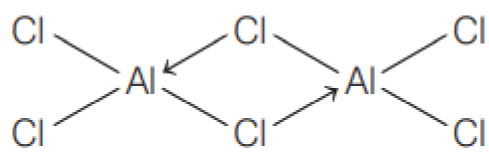
$3.57 \times v_1 = 0.02 \times 5 \times 500$

$v_1 = \frac{50}{3.57} = 14ml$

52. $v_1 = \frac{50}{3.57} = 14ml$

$Ca_3N_2 + 6H_2O \rightarrow 3Ca(OH)_2 + 2NH_3$

53. Ga_2O_3 – Amphoteric



Two Al-Cl-Al bridge bonds

81. $H_4P_2O_6$

82. Conceptual

83. The compounds of ions with charges have large lattice energies than compounds of ions with lower
Charges

84. Conceptual

85. SO_2

86. Covalent Radius<Metallic Radius

87. 19,37,55 belongs to I_A group

88. $\lambda = \frac{1}{\upsilon}$

89. $\frac{\lambda y}{\lambda x} = \frac{m_x v_x}{m_y v_y} = \frac{1}{0.1875} = 5.33$

90. Phenol (0.2% antiseptic)(1% disinfectants)

91. Mg, Sn, Zn on reaction with dil HCl liberate H_2

92. NaCN germicide, $Na_2CO_3.H_2O$ -glass

93. Conceptual

94. Lower halogen displaces higher halogens

95. $Na_2B_4O_7.10H_2O \xrightarrow{conc\ HCl} H_3BO_3 \xrightarrow{100^0 C} HBO_2 \xrightarrow{160^0 C} H_2B_4O_7$ tetraboric acid

96. $Mg_2C_3 + 4H_2O \rightarrow C_3H_4 + 2Mg(OH)_2$

97. $C_P : \overline{C} : C = 1:1.128:1.224$ if C is 12240 cm /sec then C_P is 1000cm/sec

98. Conceptual

99. $N_2 + 3H_2 \rightleftharpoons 2NH_3$

At initial 2 8 0

At equilibrium (2-x)(8-3x) (0+2x)

2-x=0.4

X=1.6

Molar concentration of $H_2 = 3.2/2 = 1.6 moles/lit$

100. $H_2PO_4^- \rightarrow H_2PO_4^{2-} + H^+$

101. Ester should be taken as principal functioning group

102. For H-COOH no ester functional isomers

103. Due to ortho effect

104. Conceptual

105. HCl does not undergo Anti markounikiffi rules

106. HCOOH beacause less steric hindrance

107. Follow the SN^1 mechanism

108. Clemenson reduction

109. $\beta -$ hydroxyl acids loses water on heating to form α, β unsaturated acids

110. $CH_3NH_2 \xrightarrow[H_2O]{HNO_2} CH_3OH \xrightarrow{PI_3} CH_3I \xrightarrow{KCN} CH_3CN \xrightarrow{LiAlH_4} CH_3CH_2NH_2$

111. $m = \dfrac{M \times 1000}{(1000 \times d) - (M \times GM\ of\ solute)} = 2.14$

112. Ortwaldi isolation methods

113. $d = \dfrac{Z \times M}{N_0 \times V}$

114. $6 \times 10^{23} electrons -------96,500 columbs$

$6.24 \times 10^{19} --------9.65 columbs$

115. $B > C > A$

116. $\pi = \dfrac{W}{GMW} \times \dfrac{ST}{V}$

117. $FeSiO_3 = slag$

118 Charge transfer phenomenon

119. Conceptual

120. Conceptual

121. $n_1T_1 = n_2T_2;\ n_2 - \dfrac{1}{4} = \dfrac{3}{4}$

$n_1 \times 300 = \dfrac{3}{4} T_2$

$T_2 = 300 \times \dfrac{4}{3} = 400K$ (or) $127^0 C$

124 . For the reaction, $H_2S \rightleftharpoons H^+ + HS^-$; $K_{a_1} = \frac{[H^+][HS^-]}{[H_2S]}$

For the reaction, $HS^- \rightleftharpoons H^+ + S^{2-}$; $K_{a_2} = \frac{[H^+][S^{2-}]}{[HS^-]}$

For the reaction, $H_2S \rightleftharpoons 2H^+ + S^{2-}$; $K_{a_3} = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$

$K_{a_1} \times K_{a_2} = K_{a_3}$

125. When the reaction is half completed $[A] = [B]$

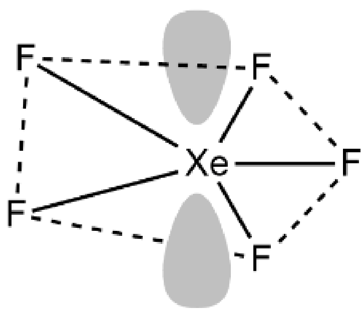
$K = \frac{[B]}{[A]} = 1$

$\therefore \Delta G^0 = -RT \ln k$

127. $Mg + H_2O \rightarrow MgO + H_2$

128. $[CCl_6]^{-2}$ does not exist because carbon has maximum valency '4'

132.



(Pentagonal planar & Non-polar)

134 . Higher the value of K_H, the lower is the solubility of gas in the liquid

135. $4LiNO_3 \rightarrow 2Li_2O + 4NO_2 + O_2$

136. $A_{(g)} \rightarrow B_{(g)} + C_{(g)}$

$p_i \quad 0 \quad 0$

$p_i - x \quad x_{atm} \quad x_{atm}$

$\therefore p_t = (p_i - x) + x + x$

$x = p_t - p_i$

Use $K = \frac{2.303}{t} \log \frac{a}{a-x}$

140. All the molecules undergoing SN^1 reaction shows racemisation as there is formation of 2 types of product in equal amount. Out of all these 4 alkyl halides , $C_6H_5CH_2Cl$ i.e

benzylic chloride has the strongest tendency to undergo SN^1 here thus leading to formation of racemic mixture as it contains benzylic carbocation which is most stable.

141. Alpha carbon of benzyl alcohol and cyclohexanol is sp^3 hybridised while in phenol and m- chlorophenol, it is sp^2 hybridised. In m-chlorophenol electron withdrawing group (— CI) is present at meta position. Presence of electron withdrawing group increases the acidic strength. So, m-chlorophenol is most acidic among all the given compounds

146. Bond order = $\frac{nb - na}{2}$

$$O_2^+ = \frac{10-5}{2} = 2.5$$

$$O_2^- = \frac{10-7}{2} = 1.5$$

$$O_2^{-2} = \frac{10-8}{2} = 1$$

$$O_2 = \frac{10-6}{2} = 1$$

Therefore, maximum bond order = minimum bond length

148. Poor screening effect of ‘d’ orbitals

149. A) Graphite has layer structure. Each layer is a planar sheet, composed of hexagonal rings of carbon atoms, with 3 electrons of each atom involved in strong single bonds with three adjacent atoms of hexagonal ring. The extra electron makes a very weak bond with the adjacent layer. Thus, one layer can slide over another and thus, it can be used as a lubricant.

B) The interlayer spacing is 3.35^0A, and not 1.42^0A. Since the layers are not joined by single bonds and 1.42^0A is the distance between two carbon atoms joined by single bond.

C) Silica is an acidic oxide and reacts only with bases and not with the acids like above.

154. $\mu = \sqrt{n(n+2)}$

160. $\Delta T_f = K_f \frac{w_2}{M_2} \times \frac{1000}{w_1}$.